Bertrand Eynard (CEA Saclay), Aisenstadt Chair

September 28 – November 6, 2015

John Harnad (Concordia)

Bertrand Eynard

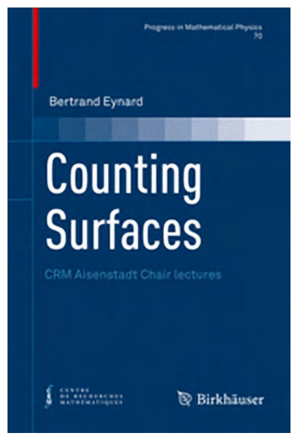
A key part of the 2015 CRM Thematic semester on AdS/CFT, Holography and Integrability, was the Aisenstadt Chair lecture series given by Bertrand Eynard (IPhT, CEA Saclay). This consisted of three lectures, the first relating to the general theme of "Topological Recursion," an area in which he has been a leading pioneer, the second and the third on the general theme of "CFT Amplitudes and Hitchin Systems." These were supplemented by two intensive series of minicourses developing the same topics in greater detail.

Bertrand Eynard is a member of the Institut de Physique Théorique, CEA Saclay. A leading innovator in a number of currently active fields of mathematical physics, including random matrices, conformal field theory, quantum gravity, string theory, graphical enumeration and combinatorics, random geometry and integrable systems, he has become known over the past decade as the driving force behind the exciting unifying development that goes under the name "Topological Recursion." This forms one of the two main topics of his 2015 Aisenstadt Chair lecture series.

He is recognized worldwide for his contributions and is a frequent invited speaker at major international meetings, including an invited talk at the 2014 ICM in Seoul, South Korea and invited plenary talks at the series of biannual String-Math conferences (most recently, in Edmonton, 2014) and the StatPhys24 conference (Cairns, Australia, 2010).

Since the founding of the CRM's Mathematical Physics Laboratory (PhysMath), Bertrand Eynard has been a valuable long-term visiting member (2001 – 2003), subsequently an external member, and a frequent collaborator with several of its regular members (M. Bertola, J. Harnad, J. Hurtubise, D. Korotkin). Throughout this time he also has been a key member of the joint FQRNT research team on Random Matrices, Moduli Spaces and Integrable Systems.

Topological recursion is an ubiquitous and universal recursive relationship between various enumerative/geometric/topological invariants associated with Riemann surfaces and their moduli, links, knots and other basic low dimensional topological invariants of current interest. Numerous applications are known in various domains of mathematics and physics, including: volumes of moduli spaces, coefficients of asymptotic expansions in random matrix theory, Hurwitz numbers, Jones polynomials, Gromov – Witten invariants, and many other combinatorial objects, all mysteriously satisfying the same recursion relations. Moreover, these relations are effective: they allow an actual computation of the invariants involved. The theory has by now been axiomatized into a definition of "new invariants" of curves. The first Aisenstadt Chair lecture, entitled *Topological recursion* (Oct. 2, 2015) introduced the notion of topological recursion, listed its properties and illustrated it with several beautiful examples. The subsequent two lectures, *CFT amplitudes and Hitchin systems* (Oct. 9 & 23, 2015) introduced a systematic construction of CFT amplitudes from an arbitrary Hitchin system, and explained the relationship between CFT conformal blocks and tau functions of integrable systems, as well as showing how the Liouville theory 4-point function is related to Painlevé VI tau function.



These Aisenstadt Chair lectures were supplemented by two intensive series of minicourses further elaborating on the topics involved. The first (Sept. 29 – Oct. 15, 2015) gave a detailed introduction to topological recursion, treating numerous examples and explaining the general formalism. The material presented forms the basis of a monograph *Counting Surfaces* published by Springer.

The first lecture dealt with two main introductory examples: Hurwitz numbers and Mirzakhani's recursion. The next gave an introduction to the general theory underlying the notion of topological recursion, introducing the associated spectral curves, both classical and quantum, and recalling some basic algebraic geometry of plane curves. The third detailed the main features of the diagrammatic computation, and introduced the notions of symplectic invariance, modular invariance, singular limits and form-cycle duality. The fourth introduced tau functions, Baker – Akhiezer functions and the Sato relations, connecting them to topological recursion. The final lecture tied together the previous constructs, relating the tau function to generating functions for the various enumerative and topological invariants associated to Riemann surfaces and their moduli spaces.

The second minicourse (Oct. 21 – Nov. 3, 2015) *Integrable Systems, Random Matrices, Hitchin Systems and CFTs* placed

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Moduli Spaces, Integrable Systems, and Topological Recursions January 9 – 13, 2016

Organizers: Dmitry Korotkin (Concordia University), Jacques Hurtubise (McGill University)

The computation and enumeration of invariants of moduli spaces took a sudden turn with the conjecture of Witten that they could be combined into a formal series that solved the KdV hierarchy. This conjecture, subsequently proven by Kontsevich, was motivated by considerations of quantum gravity. It was followed by a series of developments in the same direction, notably in the computation of invariants for Hurwitz spaces and for Gromov – Witten invariants, for example in the work of Okounkov and Pandharipande, tying Gromov – Witten theory to the 2-Toda hierarchy. Kontsevich's proof involved a detour through the theory of random matrices, and subsequently Eynard and Orantin proposed a vast generalization of the technique, with a wide variety of implications. The questions have physical motivations, and the subject has advanced with the rapid mixture of calculation and heuristic reasoning which characterizes theoretical physics; mathematicians have in many but not all cases provided proof, and, it is hoped, some understanding.

Thus a first piece of the puzzle is the theory of moduli spaces. The moduli spaces of interest are mainly associated with complex algebraic curves: moduli spaces of (pointed) curves, moduli spaces of meromorphic functions on curves (Hurwitz spaces and spaces of admissible covers) and, more generally, moduli spaces of stable maps. A second part of the puzzle, brought to the fore by Chekhov, Eynard and Orantin, has its origin in the theory of random matrices; invariants are combined into generating series from which can be computed directly a single spectral curve, by what has come to be known as the topological or Eynard – Orantin recursion. A third theme comes with the theory of Frobenius manifolds, which were introduced around 1990 by Dubrovin as a geometrization of quantum cohomology that originated from Witten – Dijkgraaf – Verlinde – Verlinde (WDVV) associativity equation in topological field theories. The quantities associated to the Frobenius manifolds are computable in terms of the topological recursion.

The theory of integrable systems seems to lie at the heart of the subject, providing a thematic link. It is fair to say, though, that the way in which it happens is still ill-understood. Indeed, so far, it is more the tools, computational devices, and actual functions that appear, rather than flows and conserved quantities.

The workshop covered all these themes, and contributed greatly to completing the picture. Thus, Hurwitz numbers of many sorts are now seen to have a computation in terms of a family of spectral curves; the same goes for the Gromov – Witten invariants of toric Calabi – Yau manifolds, where the spectral curve is in effect the mirror. The theme of topological recursion is extending its remit to knot invariants and Chern – Simons theories. Likewise, one now has a clear idea of which spectral curves correspond to Frobenius manifolds. From the integrable systems side, the ties between the topological recursion and WKB approximations for either Schrödinger operators or Hitchin systems are gradually becoming clearer. The workshop was a veritable hotbed of ideas and discussions, with lively scientific discussion not only during the talks but also between them and well into the evening afterwards. The CRM staff was in its usual efficient and pleasant form, and helped make the event a scientific success for its participants.

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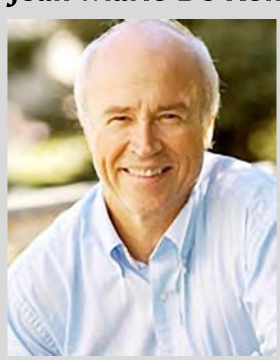
the study of conformal field amplitudes in the context of integrable systems, most notably, Hitchin systems, and the spectral theory of random matrices.

The first lecture introduced the Lax formalism, isospectral systems, algebro-geometric solutions, and Baker – Akhiezer functions, expressed in terms of prime forms and theta functions corresponding to the associated invariant spectral curves. The next lecture focused on partition functions for random matrices, spectral correlation functions and orthogonal polynomials as illustrations of solutions to integrable hierarchies. The ODEs and recursion relations satisfied by the corresponding orthogonal polynomials were shown to be isospectral or isomonodromic systems for an associated Lax matrix and meromorphic covariant derivative. The partition function and various spectral correlators were interpreted as tau functions of the associated integrable hierarchy. The last two lectures of the series dealt in greater detail with the Liouville theory 4-point function and its relation to the Painlevé VI tau function.

In January, Prof. Eynard returned to the CRM to give the opening talk of the last workshop of the thematic semester, *Moduli Spaces, Integrable Systems, and Topological Recursions.*

Prix

Jean-Marie De Koninck



Jean-Marie De Koninck

Jean-Marie De Koninck (Laval) a reçu le prix commémoratif Margaret Sinclair pour 2016 en reconnaissance de son travail de promotion des mathématiques.

Le prix commémoratif Margaret Sinclair souligne l'innovation et l'excellence dans l'enseignement des mathématiques au niveau élémentaire, secondaire, collégial et universitaire.