

# On the hyperbolic lattice point problem

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The general problem is to estimate the number of points of the orbit  $\Gamma z$  inside a disc of increasing radius, where  $\Gamma$  acts discontinuously on a metric space  $X$ . For the Euclidean disc this relates to representations as sum of two squares. There is a long history for this problem and a well-studied conjecture of Hardy for the error term. For hyperbolic space and cofinite groups e.g.  $SL(2, \mathbb{Z})$  Selberg proved an error term  $O(X^{2/3})$  (unpublished), which has never been improved for a single group. Various other mathematicians worked on this: Huber, Patterson, and Phillips-Rudnick, who first showed a lower bound for the error term. All results lead to conjecture an error of order  $O(X^{1/2+\varepsilon})$ . I will explore various averages and lower bounds in this problem for  $SL(2, \mathbb{Z})$ , using results that depend strongly on arithmeticity, via the study of Maass cusp forms.

I will report also on another variation of this problem, due to Huber, where the new estimates on the error term use the large sieve for  $\Gamma H$ , first studied by Chamizo.

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