Conférence
« ARTA III : Avancées en théorie des représentations des algèbres »
16-20 juin 2014

Meeting
"ARTA III. Advances in Representation Theory of Algebras"
June 16–20, 2014

Tilting theory for one dimensional hypersurfaces

Kota Yamaura*

One of important subjects in recent representation theory is to compare two major classes of triangulated categories, which are the stable categories and the derived categories. In my talk, we consider the question "*when is the stable category* $\underline{\mathrm{CM}}^{\mathbb{Z}}(R)$ *of graded Cohen—Macaulay modules over a graded Gorenstein commutative ring* R *triangle equivalent to the derived category of a ring*".

The answer to the question was given in several cases, for example, R is of Krull dimension zero [**?**], R is one dimensional and finite CM representation type [**?**], R is a quotient isolated singularity [**?**]. In their study, tilting theory for triangulated categories played a key role.

Our aim is to study the question for the case that R is a one dimensional hypersurface with the standard grading. First we find a tilting object in the stable category $\underline{\mathrm{CM}}^{\mathbb{Z}}_0(R)$ of $\mathbb{Z}$-graded Cohen—Macaulay R-modules which are locally free at the punctured spectrum.

Theorem 1. *There exists a tilting object* V *in* $\underline{\mathrm{CM}}^{\mathbb{Z}}_0(R)$.

By Theorem 1 and tilting theorem (cf. [**?**]), we have a triangle equivalence

$$\underline{\mathrm{CM}}^{\mathbb{Z}}_0(R) \simeq \mathrm{K}^{\mathrm{b}}(\mathrm{proj}\Gamma).$$

where $\Gamma := \mathrm{End}_{\underline{\mathrm{CM}}^{\mathbb{Z}}_0(R)}(V)$. Next we extend this to the triangle equivalence stated below. Then we have an answer to the question for our case.

Theorem 2. *There exists a triangle equivalence*

$$\underline{\mathrm{CM}}^{\mathbb{Z}}(R) \simeq \mathrm{D}^{\mathrm{b}}(\mathrm{mod}\Gamma).$$

*Faculty of Engineering, Yamanashi University, 4-3-11, Kofu Takeda, 400-8511, JAPAN.

As applications of the above results, we have various triangle equivalences between the stable categories and the derived categories / homotopy categories.

Joint work with Martin Herschend, Osamu Iyama and Ryo Takahashi.

References

[1] T. Araya, *Exceptional sequences over graded Cohen-Macaulay rings*, Math. J. Okayama Univ. 41 (1999), 81--102 (2001)

[2] O. Iyama, R. Takahashi, *Tilting and cluster tilting for quotient singularities*, Math. Ann. 356 (2013), no. 3, 1065--1105.

[3] B. Keller, *Deriving DG categories*, Ann. Sci. Ecole Norm. Sup. (4) 27 (1994), no. 1, 63--102.

[4] K. Yamaura, *Realizing stable categories as derived categories*, Adv. Math. 248 (2013), 784--819.