

# Spectral transformations, CMV matrices and integrable systems

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The so-called Darboux factorization of Jacobi matrices, which are the canonical representations of self-adjoint operators, provides, among other applications, a solution of the discrete bispectral problem. This factorization is also related with certain modifications of the corresponding orthogonality measure, the so called linear spectral transformations ([4]).

A natural question is to analyze this situation for the canonical representations of unitary operators. The Darboux factorization of CMV matrices can be useful to study similar problems on the unit circle.

In this lecture we relate Cholesky factorizations, CMV matrices and certain Laurent perturbations of measures on the unit circle. These factorizations not only gives a direct connection between the corresponding CMV matrices but also yield the relation between the corresponding sequences of orthonormal Laurent polynomials (see [2]). We will focus our attention on an integrable system related to the Uvarov transformation on the unit circle (see [3] in the case of the real line).

Finally, we will deal with the Delsarte-Genin map from orthogonal polynomials on the unit circle to symmetric polynomials on the interval  $[-1, 1]$  as an alternative way to the standard Szegő map. A connection with the results presented in [1] will be stated.

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## References

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