École
« L'abécédaire de SIDE (ASIDE) »
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School
"Abecedarian of SIDE (ASIDE)"
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SESSION—Discrete integrable systems and isomonodromy transformations. Yang-Baxter maps and quantum discrete integrable systems

Sotiris Konstantinou-Rizos, Chechen State University
Deniz Bilman, University of Michigan

Titre/Title : "Darboux & Bäcklund transformations: Discrete systems, matrix refactorisation problems and Yang-Baxter maps" by *Sotiris Konstantinou-Rizos*

In this lecture, we give an introduction to Darboux and Bäcklund transformations. We show how Darboux transformations of (integrable) PDEs can be used to construct integrable discrete systems, namely we present the so-called "Lax-Darboux" scheme.

Moreover, we give an introduction to the theory of Yang-Baxter maps, namely set-theoretical solutions of the quantum Yang-Baxter equation. We explain the relation between Yang-Baxter maps and discrete integrable systems on quad graphs. We present a method of constructing Yang-Baxter maps using Darboux transformations, and show how the first integrals of the associated Bäcklund transformation can be used to reduce these YB maps to completely integrable maps in the Liouville sense.

As an illustrative example, we derive the Darboux transformation of the nonlinear Schrödinger (NLS) equation. Via this Darboux transformation, the NLS equation is related to a discrete Adler-Yamilov type of system, as well as the discrete Toda equation. We also consider the matrix refactorisation problem of the associated NLS Darboux matrix, in order to construct a Yang-Baxter map, which can be restricted on invariant leaves to the Adler-Yamilov Yang-Baxter map, which is completely integrable. Finally, we briefly review some recent results related to NLS type equations.

Titre/Title : "The Toda lattice" by *Deniz Bilman*

In this lecture, we consider the famed doubly-infinite Toda lattice and present the inverse scattering transform method for the solution of the Cauchy initial value problem. We begin with reviewing properties of solutions and existence of solitons. As is well known, the Toda lattice equations can be recast as an isospectral flow on Jacobi matrices and this gives rise to the existence of a Lax pair. Thus we move on to cover scattering theory for Jacobi matrices, introduce the scattering transform and scattering data associated with a Jacobi matrix. Then we cover the time evolution of the scattering data under the dynamics induced by the Toda lattice equations and present the Riemann-Hilbert formulation of the inverse scattering transform. We review some results on long-time asymptotics of the solutions of the Cauchy initial problem for sufficiently decaying initial data.