

Heavenly constructions of quaternionic manifolds (following LeBrun, Feix and Kaledin)

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In 1982, Claude showed how to obtain selfdual Einstein metrics from conformal 3-manifolds. His method constructs explicitly a complex contact 3-manifold containing “special” (in fact legendrian) twistor lines (rational curves with normal bundle a sum of $O(1)$'s). He then showed that the Kodaira moduli space of real (non-Legendrian) deformations of the special twistor lines is a selfdual Einstein 4-manifold with the original 3-manifold “at infinity”.

Nearly 20 years later, Birte Feix and Dmitry Kaledin independently obtained a general existence result for hyperkaehler metrics on contangent bundles. Feix's method also proceeds by constructing explicitly a complex manifold with special twistor lines, and showing that the Kodaira moduli space of their real deformations carries a $U(1)$ -invariant hyperkaehler metric with the original Kaehler manifold as a fixed submanifold of the $U(1)$ action.

In this talk I discuss a correspondence between $U(1)$ -invariant quaternionic manifolds with a maximal totally complex fixed submanifold, and complex manifolds with a so-called c -projective structure. This generalizes and provides a natural context for the results of Feix and Kaledin. In the four dimensional case, it is also related to a minitwistor construction of asymptotically hyperbolic Einstein—Weyl structures, a concept also introduced and studied in Claude's work.

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