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Bounded, non-vanishing solutions of KdV and a new class of potentials of the Schroedinger operator

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The KdV hierarchy is an infinite collection of commuting isospectral deformations of the one-dimensional Schroedinger operator, and its spectral theory is related to the initial-value problem for KdV. For two classes of initial data, the spectral theory is well understood, and the initial value problem can be considered solved. A potential rapidly vanishing at infinity can be reconstructed from its spectral data by using the inverse spectral transform (IVT), and the spectral data evolves linearly with KdV. An important class of such potentials are the Bargmann potentials, or soliton solutions of KdV. The spectrum of a periodic potential consists of an infinite sequence of bands separated by spectral gaps. For a dense collection of potentials, there are only finitely many gaps, the eigenfunction is identified as a section of a line bundle over a corresponding hyperelliptic curve, and the KdV evolution is linear on the Jacobian of the curve.

It has long been known that finite-gap potentials should be obtainable as limits of Bargmann potentials, but a precise description of such a limit was not known. We reformulate the IVT by studying the singularities of the eigenfunctions of the corresponding Schroedinger operator, which gives us some additional freedom for describing the Bargmann potentials. Replacing the isolated singularities with cuts on the spectral plane, we obtain a new Riemann—Hilbert problem whose solutions describe potentials of the Schroedinger operator that are non-vanishing at infinity, but are not periodic, and can be thought of as a one-dimensional soliton gas. This RH problem can be studied numerically, and we also study the spectra of the corresponding Schoedinger operators.

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