

Tail-Weighted Dependence Measures and Estimation of Joint Tail Probabilities

Harry Joe *

harry.joe@ubc.ca

Based on pairwise normal scores plots and diagnostics of tail-weighted dependence measures, one can get a good idea if variables have upper or lower tail dependence. When comparing fits of different parametric copula models based on log-likelihoods and AIC/BIC values, the best-fitting models typically have tail dependence if this is seen in the diagnostics. How reliable are joint tail probabilities as functions of fitted parametric copula models? It depends on how far one is in the tail, and the limiting tail dependence coefficients are especially not reliable based on parametric copula models. Similar to inference for univariate extreme quantiles, one must be careful in how one estimates tail quantities. Because tail dependence coefficients cannot be directly estimated, tail-weighted dependence measures have been used for data summaries and assessment of adequacy of model fits. A new family of dependence measures, ζ_α for $\alpha > 0$, is presented that has the following properties: (a) for $\alpha = 1$, ζ_α is a measure of central dependence, (b) for large α , ζ_α is a tail-weighted dependence measure, (c) as $\alpha \rightarrow \infty$, ζ_α is the upper tail dependence coefficient. With an expansion of ζ_α over large α , we can determine some conditions for which one might have a reliable non-parametric estimate of the tail dependence coefficient. Examples will be used for illustration of the main ideas.

*Department of Statistics, University of British Columbia, 3182 Earth Sciences Bldg, 2207 Main Mall, Vancouver, BC V6T 1Z4, CANADA