

# The Noncommutative Geometry of Difference Equations

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One can view the theory of  $D$ -modules (on curves) as a noncommutative analogue of the theory of sheaves on the cotangent bundle, extending the usual relaxation of connections on vector bundles to Higgs bundles. It turns out that (after a suitable compactification) this extends considerably: any rationally ruled projective surface admits a natural family of noncommutative deformations equipped with interpretations of certain sheaves as (discrete) connections (e.g., elliptic difference equations, the Higgs-ish relaxation of which was studied by Hurtubise and Markman). I'll discuss a number of aspects of this construction, with particular attention to structural properties of the moduli spaces (projectivity, Poisson structures), associated nonautonomous integrable systems, and certain derived actions of  $SL_2(\mathbb{Z})$  on deformations of elliptic surfaces.

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