

Estimation in the presence of group actions

Alex Wein*

awein@mit.edu

Imagine we want to recover an unknown vector given many noisy copies of it, except each copy is cyclically shifted by an unknown offset (this is “multi-reference alignment”). Or imagine we want to reconstruct an unknown 3D structure (e.g. a molecule) given many noisy pictures of it taken from different unknown angles (this is “cryo-EM”). These problems (and many others) belong to the class of “orbit recovery” problems wherein we observe many noisy copies of an unknown vector, each acted upon by a random element of some compact group. I will show how to determine the optimal sample complexity of such problems, which is achieved by the method of moments (or equivalently, the method of invariants). The analysis involves tools from invariant theory and algebraic geometry. I will also discuss issues of computational efficiency.

This is based on joint work with Afonso Bandeira, Ben Blum-Smith, Amelia Perry, and Jonathan Weed (<https://arxiv.org/abs/1712.10163>).

*Department of Mathematics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA