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Superintegrable systems with higher order integrals of motion and the Painlevé transcsendents

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A review is given of families of superintergrable classical and quantum systems in two-dimensional Euclldean space that allow separation of variables in Cartesian or polar coordinates and allow an additional integral of motion. This integral is assumed to be a polynomial of order $N > 2$ in the momenta. These families include systems with Hamiltonians expressed in terms of elementary functions but also families of potentials expressed in terms of solutions of nonlinear differential equations, We conjecture that the equations have the Painleve property for all N. So far this has been proven for $N = 3$, 4 and 5. All maximally superintegrable systems share the properties of the best known ones, namely the Kepler-Coulomb and harmonic oscillator ones. In particular in classical mechanics all bounded trajectories are closed. In quantum mechanics all bound state energies are degenerate and the Schrodinger equation is conjectured to be exactly solvable.

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