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Invited lecture - Special Session in honour of Decio Levi
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$\mathbb{C}P^{2S}$ sigma models described through hypergeometric orthogonal polynomials

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The main objective of this talk is to establish a new connection between the Hermitian rank-1 projector solutions of the Euclidean $\mathbb{C}P^{2S}$ sigma model in two dimensions and the particular hypergeometric orthogonal polynomials called Krawtchouk polynomials. We show that any such projector solutions of the $\mathbb{C}P^{2S}$ model, defined on the Riemann sphere and having a finite action, can be explicitly parametrised in terms of these polynomials. We apply these results to the analysis of surfaces associated with $\mathbb{C}P^{2S}$ models defined using the generalised Weierstrass formula for immersion. We show that these surfaces are homeomorphic to spheres in the $\mathfrak{su}(2s+1)$ algebra, and express several other geometrical characteristics in terms of the Krawtchouk polynomials. Finally, a connection between the $\mathfrak{su}(2)$ spin-s representation and the $\mathbb{C}P^{2S}$ model is explored in detail. It is shown (Proposition 5.2) that for any given holomorphic vector function in $\mathbb{C}^N$, it is possible to derive solutions of the $\mathbb{C}P^{2S}$ model through algebraic recurrence relations which turn out to be simpler than the analytic relations known from the literature.

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