

Contravariantly finite subcategories closed under predecessors

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The results we are going to discuss here are part of a joint work with I. Assem (Sherbrooke) and S. Trepode (Mar del Plata).

Let A be an artin algebra and \mathcal{C} be a subcategory of the category of finitely generated A -modules $\text{mod}A$ which is closed under predecessors. The main aim of this talk is to give some equivalent conditions to \mathcal{C} being contravariantly finite. Recall that a subcategory \mathcal{X} of $\text{mod}A$ is *contravariantly finite* provided for each $M \in \text{mod}A$, there exists a morphism $f_M: X_M \rightarrow M$ with $X_M \in \mathcal{X}$ such that any morphism $f: X \rightarrow M$ with $X \in \mathcal{X}$ factors through f_M . One of our main result reads as follows. Denote by E the sum of all indecomposable Ext-injective modules of \mathcal{C} (which is a finitely generated A -module by a result by Assem).

THEOREM. *Let \mathcal{C} be a full subcategory of $\text{ind}A$, closed under predecessors. The following conditions are equivalent:*

- (a) *$\text{add } \mathcal{C}$ is contravariantly finite.*
- (b) *\mathcal{C}^\perp is covariantly finite.*
- (c) *E is a cotilting ${}_{\mathcal{C}}A$ -module.*
- (d) *$\text{add } \mathcal{C} = \text{Supp}(-, E)$.*
- (e) *Any morphism $f: L \rightarrow M$ with $L \in \mathcal{C}$ and M indecomposable not in \mathcal{C} factors through $\text{add}E$.*