Colloque CRM-ISM Colloquium
14 December 2007

Smale's 17th Problem

Michael Shub
Department of Mathematics
University of Toronto
40 St George Street
Toronto, Ontario M5S 2E4
CANADA
[michael.shub@utoronto.ca]

Abstract

In a series of papers written in the first half of the 1990's Steve Smale and I studied the complexity of solving systems of n polynomial equations in n complex variables. We studied path following techniques. A system with known solution is connected by a path to the system we want to solve and the solution is "continued" along the path. The path we chose was the straight line connecting the systems. We proved that "on average" systems can be solved with polynomial cost but we did not prove the existence of a uniform algorithm. The question of the existence of a uniform algorithm is Smale's 17th problem. Recently, Beltran and Pardo have made significant progress on this problem. Moreover, Jointly with Beltran I have linked the complexity to the length of the (problem,solution) path in the condition number Riemannian structure. Surprisingly short paths exist!