Homogeneity and algebraic closure in free groups

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The first part of the talk deals with homogeneity of free groups and prime models. We show that any free group of finite rank is homogeneous, that is if two tuples have the same type then they correspond each other by an automorphism. We also show that any non-free two-generated torsion-free hyperbolic group is existentially homogeneous and prime. This gives in particular examples of prime groups which are not QF and answers a question of A. Nies.

The second part deals with the algebraic and definable closure (acl and dcl) in free groups and it is a joint work with my student D. Vallino.

We prove that if F is a free group of finite rank and A is a nonabelian subgroup of F such that F is freely indecomposable with respect to A, then $\mathrm{acl}(A)$ is exactly the vertex group in the cyclic JSJ-decomposition of F with respect to A.

We show that $\mathrm{dcl}(A)$ is a free factor of $\mathrm{acl}(A)$ and in particular they coincide in a free group of rank 2. In the general case, we show that a free group whose rank is greater than 4 contains a subgroup A such that $\mathrm{acl}(A) \neq \mathrm{dcl}(A)$. This answers a question of Z. Sela.

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