On the long-time behavior of solutions of 2-d Euler equations

Alexander Shnirelman

Department of Mathematics & Statistics Concordia University 1455 de Maisonneuve Blvd. W. Montréal (Québec), H3G1M8 CANADA shnirel@mathstat.concordia.ca

Abstract

The problem of long-time behavior of 2-d flows of an ideal incompressible fluid is a "junior sister" of the celebrated Millenium Problem regarding the possibility of singularity in a solution of 3-d Euler (or Navier–Stokes) equations in finite time, and can give some clues to the last problem. Experiments and numerical simulations indicate that generic 2-d flows tend to some steady and stable solution of the Euler equation as $t \to \infty$. Such behavior is in apparent contradiction with the fact that the ideal incompressible fluid, described by the Euler equations, is a conservative system, and the Euler equations are timereversible. In fact, there is no contradiction, but understanding this paradox requires a deeper insight into the nonequilibrium statistical mechanics and infinite-dimensional differential geometry. The talk is devoted to the experimental and partial rigorous results around this paradox.