"Integrable quantum systems and solvable statistical mechanical models"
«Systèmes quantiques intégrables et modèles statistiques résolubles»
June 30 – July 5, 2008/**30 juin – au 5 juillet 2008**

Behavior of the two-dimensional Ising model at the boundary of a half-in nite cylinder

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Abstract

The two-dimensional Ising model is studied at the boundary of a half-in nite cylinder. The three regular lattices (square, triangular and hexagonal) and the three regimes (sub-, super- and critical) are discussed. The probability of having precisely $2n$ spinflips at the boundary is computed as a function of the positions $k_i's$, $i = 1, \ldots, 2n$; of the spinflips. The limit when the mesh goes to zero is obtained. For the square lattice, the probability of having $2n$ spinflips, independently of their position, is also computed. These results are obtained as consequences of Onsager's solution and are rigorous. In the special case of precisely 4 spinflips, we use conformal field theory to give a prediction for the following probability. Let θ_1; θ_2; θ_3 and θ_4 be the positions of the flips along the boundary. We give the probability distribution that the contour leaving θ_1 ends at θ_2 instead than at θ_4. The behavior of this function when $\theta_2 - \theta_1 \to 0$ is described by a power law with an exponent $(\frac{5}{3})$ that belongs to the Kac table but that corresponds to a non-unitarizable highest-weight representation. We check that this prediction agrees with a Monte-Carlo simulation.

Joint work with Louis-Pierre Arguin and Hassan Aurag, Centre de Recherches Mathématiques, Université de Montréal.