On the motivic cohomology of Severi–Brauer varieties

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Abstract

We construct a spectral sequence converging to the motivic cohomology of a Severi–Brauer variety X, whose second term coincides with the motivic cohomology of the Voevodsky simplicial scheme $C(X)$. In case of a trivial Severi–Brauer variety we recover the usual decomposition of the motivic cohomology of the projective space as a direct sum of several copies of the motivic cohomology of the base field. A really complete answer may be obtained in case of algebras of prime degrees since in this case motivic cohomology of $C(X)$ may be completely domputed (this computation was performed independently by A. Merkurjev). As an application we show that reduced norm map $\mathrm{Nrd}\colon K_2(D) \to K_2(F)$ is injective for algebras of prime degree. We also try to give a cohomological computation of groups $\overline{K}_2(D)$ and $SK_2(D)$ for a biquaternion algebra.