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Coloring Number and On-line Ramsey Theory for Graphs and Hypergraphs

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Let c, s, t be positive integers. The (c, s, t)-*Ramsey game* is played by Builder and Painter. Play begins with an s-uniform hypergraph $G_0 = (V, E_0)$, where $E_0 = \emptyset$ and V is determined by Builder. On the ith round Builder constructs a new edge e_i (distinct from previous edges) and sets $G_i = (V, E_i)$, where $E_i = E_{i-1} \cup \{e_i\}$. Painter responds by coloring e_i with one of c colors. Builder wins if Painter eventually creates a monochromatic copy of K_s^t, the complete s-uniform hypergraph on t vertices ; otherwise Painter wins when she has colored all possible edges.

We extend the definition of coloring number to hypergraphs so that $\chi(G) \leq \mathrm{col}(G)$ for any hypergraph G and then show that Builder can win (c, s, t)-Ramsey game while building a hypergraph with coloring number at most $\mathrm{col}(K_s^t)$. An important step in the proof is the analysis of an auxiliary *survival game* played by Presenter and Chooser. The (p, s, t)-*survival game* begins with an s-uniform hypergraph $H_0 = (V, \emptyset)$ with an arbitrary finite number of vertices and no edges. Let $H_{i-1} = (V_{i-1}, E_{i-1})$ be the hypergraph constructed in the first $i-1$ rounds. On the i-th round Presenter plays by presenting a p-subset $P_i \subseteq V_{i-1}$ and Chooser responds by choosing an s-subset $X_i \subseteq P_i$. The vertices in $P_i - X_i$ are discarded and the edge X_i added to E_{i-1} to form E_i. Presenter wins the survival game if H_i contains a copy of K_s^t for some i. We show that for positive integers p, s, t with $s \leq p$, Presenter has a winning strategy.

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