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Tracking propagation of self-intersecting fronts: an invariant manifold approach

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Abstract

In many front propagation problems, the front's position at the time t is represented as a particular level set of a solution to the corresponding first-order non-linear PDE. Typically, this PDE does not have a globally smooth solution and a unique (single-valued) viscosity solution is sought to automatically remove emerging self-intersections of the front. However, a smooth multi-valued solution is needed instead whenever the behavior of characteristics is still relevant even after their intersection (e.g., in geometric optics, multiple-arrival seismic imaging, and tomography).

Multi-valued solutions are frequently computed using Lagrangian methods (e.g., ray tracing)—solving the ($2n$-dimensional system of) characteristic ODEs for a finite number of initial conditions determined by the boundary conditions of the original PDE. The disadvantage of this approach is a poor spatial resolution due to a non-uniform rate of separation for trajectories of the ODE system. A number of Eulerian methods were proposed to alleviate this problem by solving a corresponding Liouville system of n linear PDEs in a $2n$-dimensional (x, p) phase space.

We propose an alternative approach, which restricts the computations to an n-dimensional mesh approximating a particular invariant manifold of the characteristic vector field. That manifold is locally modeled as a graph of the function satisfying a quasi-linear flow-invariance PDE, which is then efficiently solved by a space-marching method in Eulerian framework. We illustrate our approach by computing multi-valued solutions for several Hamilton-Jacobi PDEs and hyperbolic conservation laws.

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