Exact Internal Control of Nonlinear Schrödinger Equations

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Abstract.

We consider the exact internal control problem for a model nonlinear Schrödinger equation of type

$$\begin{cases} iu_t = -u_{xx} - |u|^2 u + c_U h \\ u(0,x) = u_0(x), u(T,x) = u_1(x) \end{cases} \qquad (0 \le x \le L, 0 \le t \le T) \tag{1}$$

together with either homogeneous Dirichlet or periodic boundary data on $[0, L]$; u_0 (the *initial function*) and u_1 (the *target function*), and $T > 0$ are given; furthermore, U is a given fixed open subset of $[0, L]$ with characteristic function c_U.

There are applications of problems of this type in nonlinear optics and laser physics (e.g. the control of 1-cycles in the 'ring cavity problem'), or in *quantum control* of chemical systems by laser devices.

We show that problem (1) is exactly internally controllable on any U if the data u_0 and u_1 and are in $H^1(0, L)$. The method consists of an application of Schauder's fixed point theorem to an appropriate convex subset K of the space $C\big([0, T], L^2(0, L)\big)$; the fixed point operator F is defined by $F(u) := w_u$ (for $u \in K$) where $w = w_u$ is a solution of the linear exact internal control problem

$$\begin{cases} iw_t = -w_{xx} - |u|^2 w + c_U h_u \\ w(0,x) = u_0(x), w(T,x) = u_1(x) \end{cases} \qquad (0 \le x \le L, 0 \le t \le T) \tag{2}$$

with an appropriate control function h_u; (2) is solved by using the Hilbert-Uniqueness-Method introduced by J. L. Lions. The results can be generalized to more general problems of type (1) (e.g. with nonlinearities of type $f(|u|)^2$) and to higher dimensions.

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