On weak holonomy

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Abstract

The subject of this talk is the notion of weak holonomy group, defined by A. Gray. We prove that $\mathrm{SU}(n)$ with $n > 2$ and $\mathrm{Sp}(n)\mathrm{U}(1)$ with $n > 1$ are the only connected Lie groups acting transitively and effectively on some sphere which can be weak holonomy groups of a Riemannian manifold without having to contain its holonomy group. In both cases the manifold is Khler. This implies, in particular, that the nearly Khler manifolds do not have weak holonomy $\mathrm{SU}(n)$ or $\mathrm{U}(n)$ and that the manifolds with nearly parallel vector cross product do not have weak holonomy G2.