Fluid limit of a network with fair bandwidth sharing and general document size distributions

Christian Gromoll
EURANDOM
5612 AZ Eindhoven, The Netherlands

Abstract

Roberts and Massoulié (1998) have introduced a model of Internet congestion control that represents the randomly varying number of flows in a network where bandwidth is dynamically shared among flows. Flows correspond to continuous transfers of individual documents through the network. Thus, the model assumes a "separation of time scales" between flow level dynamics and packet level dynamics in the network; the time scale of document arrivals and departures (flow level) is much longer than the time scale on which rate control schemes such as TCP converge to equilibrium (packet level).

This work proposes a fluid model, or formal functional law of large numbers approximation, for the flow level model operating under a weighted α-fair bandwidth sharing policy, introduced by Mo and Walrand (2000). This policy offers a generalization of the processor sharing discipline from a single resource to a network with several shared resources. In contrast to earlier work of Kelly and Williams (2003), the present fluid model allows general, rather than exponentially distributed, interarrival times and document sizes. To describe the evolution of the system, measure valued processes are used to keep track of the residual sizes of all documents in the system at any given time. Under mild conditions, the appropriately rescaled measure valued processes corresponding to a sequence of flow level models (with fixed network structure) are tight, and each weak limit point of the sequence is almost surely a fluid model solution.

Joint work with Ruth Williams