Spatial Models for Ad Hoc Wireless and Sensor Networks

Optimizing for Energy Efficiency

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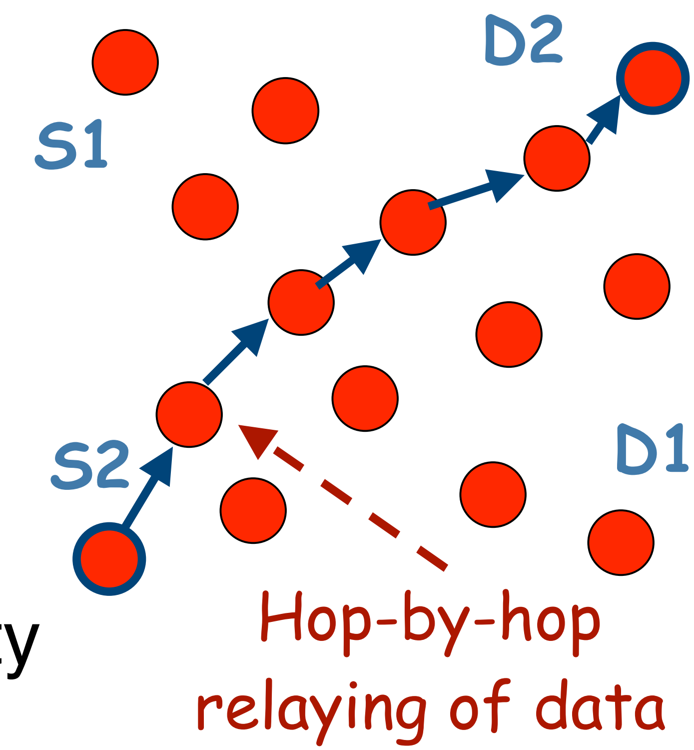
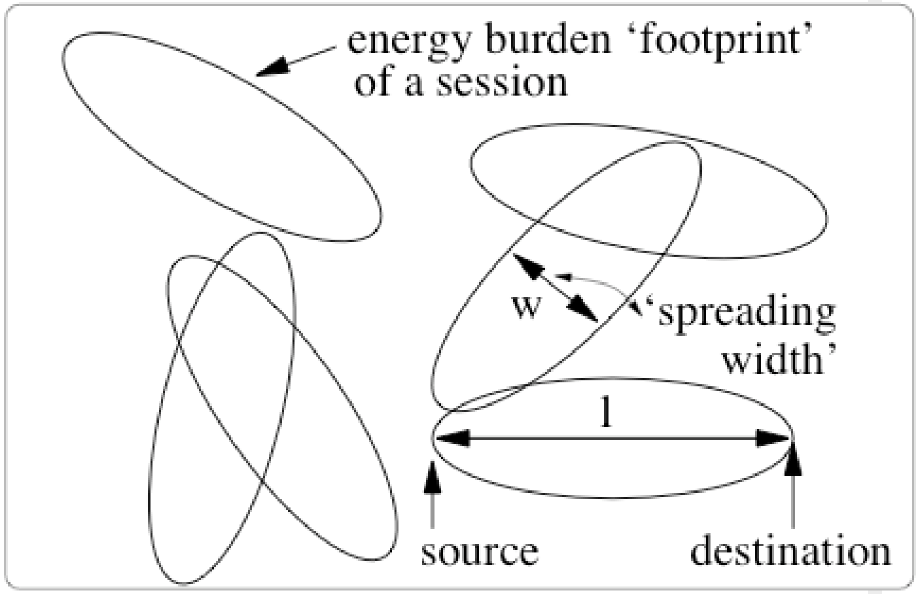
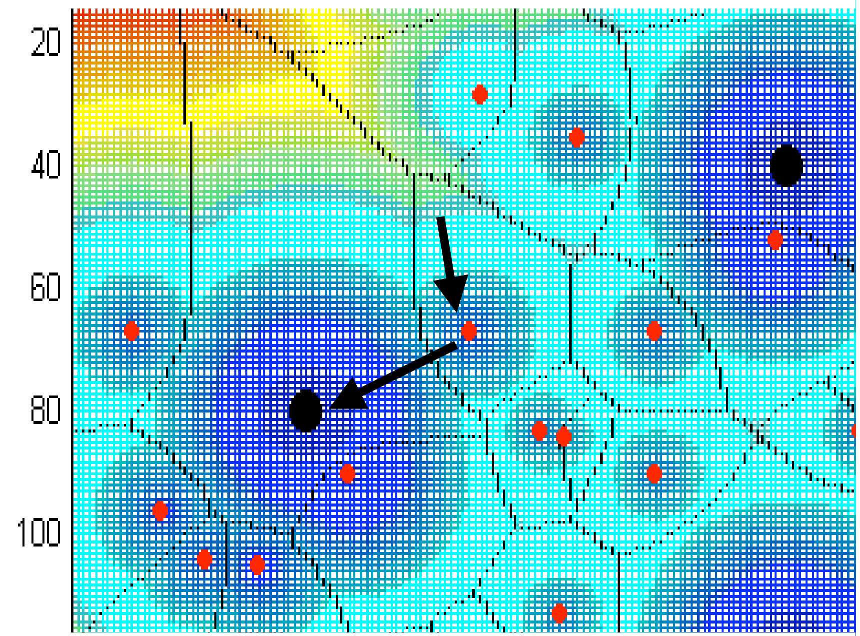
Motivation - Ad hoc and Sensor Nets

- Distributed peer-to-peer networking and/or sensing applications enabled by local wireless communication links
- **Energy burdens**
  - Computation
  - **Communication**
- Typically limited
  - Battery reserves
  - Replenishing capability



This Talk: Optimal `shapes' for energy sensitive routing & hierarchies





- Balancing energy burdens by spreading traffic loads versus increased energy costs to realize spreading
- Decrease energy burdens via data/header compression versus energy cost to reach compression nodes.

Two tradeoffs to be explored

Talk Outline

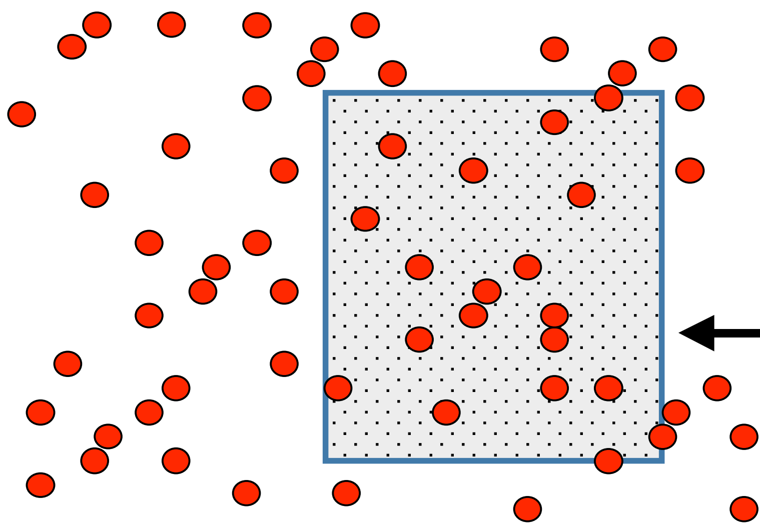
- Some background on stochastic geometry
  - See e.g., Moeller, Kendall, Stoyan & Mecke
  - Telecom Models: Baccelli, Zuyev, and collaborators

- Part 1: Routing for energy balancing in ad hoc wireless networks

- Part 2: Routing hierarchies for wireless sensor networks using compression and sink nodes

Poisson Point Process with intensity λ

- Modeling spatial traffic loads and/or locations of network/sensor nodes



Point Process

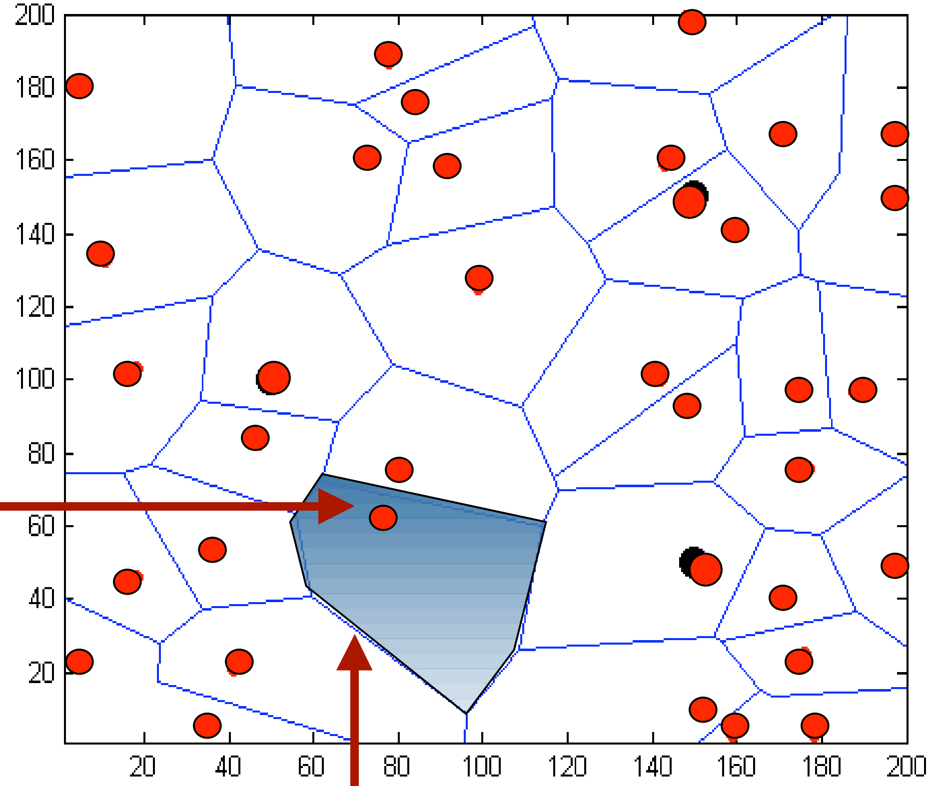
$$\Pi = \{X_i | i = 1, 2, 3..\}$$

A : region with area |A|

$$N(A) = \sum_{X_i \in \Pi} \mathbf{1}(X_i \in A) \sim \mathrm{Poisson}(\lambda |A|)$$

Voronoi Tesselation induced by Π=π

- Modeling spatial network/routing hierarchies

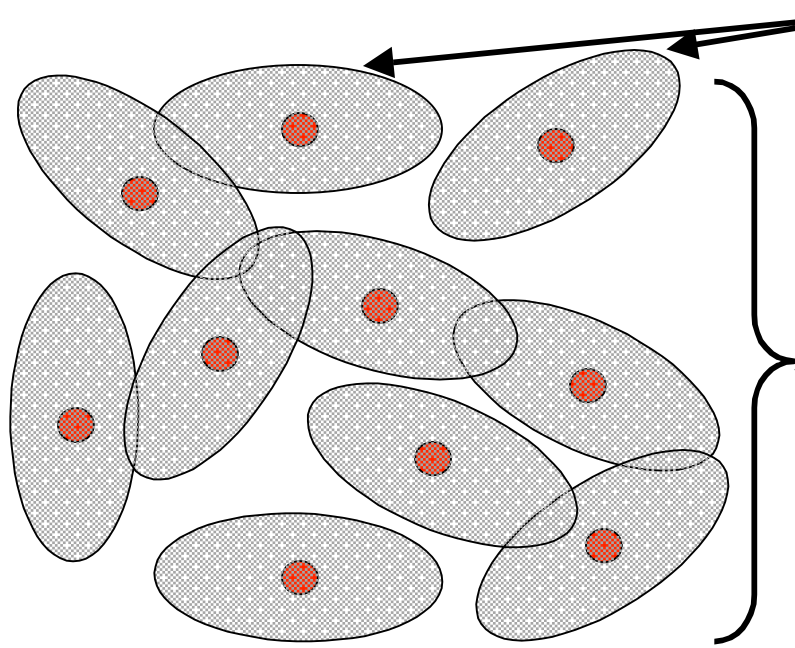


x_i

$$V_{x_i}(\pi) = \{z \in \mathbf{R} | |z - x_i| < |z - x_j|, \forall x_j \in \pi\}$$

Boolean Model

- Modeling random sets: e.g., coverage of wireless service/sensors



i.i.d random sets/shapes

$$\{\Phi_i | i = 1, 2, \ldots\}$$

$$\Phi_i \sim \Phi_0, i = 1, 2, \ldots$$

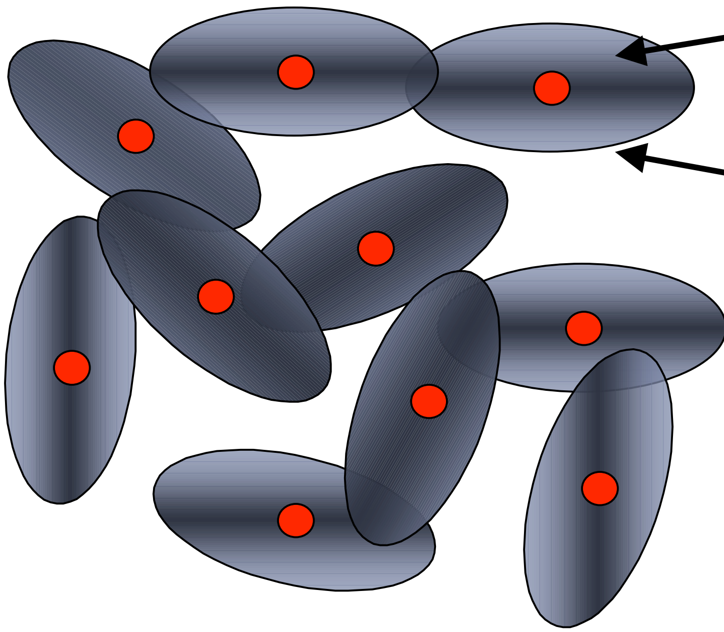
Boolean model: random set

$$\cup_{X_i \in \Pi} \Phi_i \oplus \{X_i\}$$

Shifts set to X_i

Shot-Noise Process

- Modeling spatial fields: e.g., traffic overlaps spatial energy burdens induced by routing



`Random' functions :

$$h(\cdot, \Phi_i) : \mathbf{R}^2 \rightarrow \mathbf{R}$$

support set

Shot-Noise process:

$$E(x) = \sum_{X_i \in \Pi} h(x - X_i, \Phi_i)$$

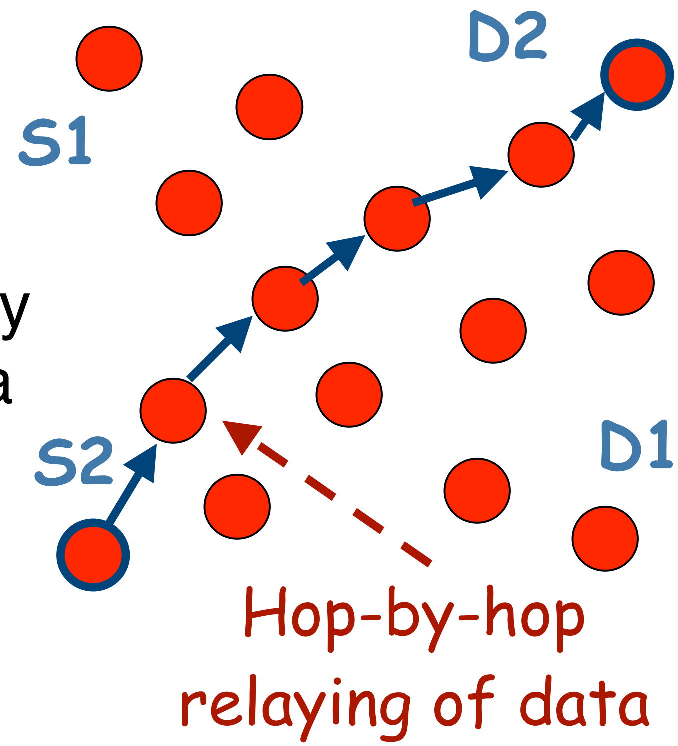
shifts h(,) to X_i

Example:

$$E(x) = \sum_{X_i \in \Pi} \mathbf{1}(x \in \Phi_i \oplus X_i) \sim \text{Poisson}(\lambda E[|\Phi_0|])$$

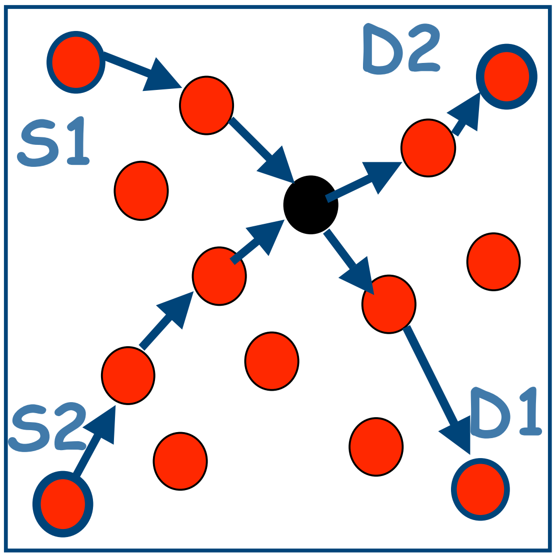
Part 1: Routing for energy balancing in ad hoc wireless networks

- Simple routing/energy model
  - Hop-by-hop routing along `neighboring' nodes
  - Same transmit/receive energy expenditure per hop/unit data
  - Energy expenditure proportional to data flow rate and # of hops.

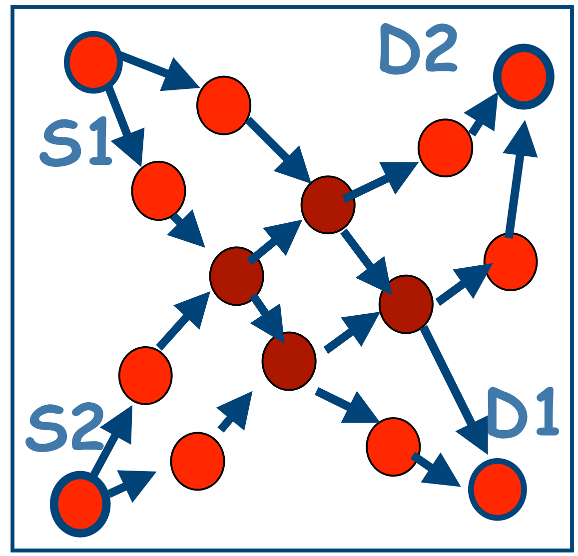


Energy Balancing - Multipath Routing

Shortest Path Routing



Multipath Routing



Unbalanced energy burdens `Balanced' energy burdens

- Poor `balancing' of energy burdens results in
  - Energy hotspots with eventual total depletion of node's energy reserves.
  - Possibly use of inefficient longer routes to circumvent depleted areas (future work)

Related work

- Dynamic shortest path routing based on depletion levels
  - Overheads (updating state) and robustness
- `Optimal' dynamic multipath routing to extend network liftetime [Chang and Tassiulas]
  - Overheads and scalability
- Randomized packet routing to spread loads across fixed region in a grid [Servetto and Barrenechea]
  - Randomization energy efficient ?
  - How much should one spread?

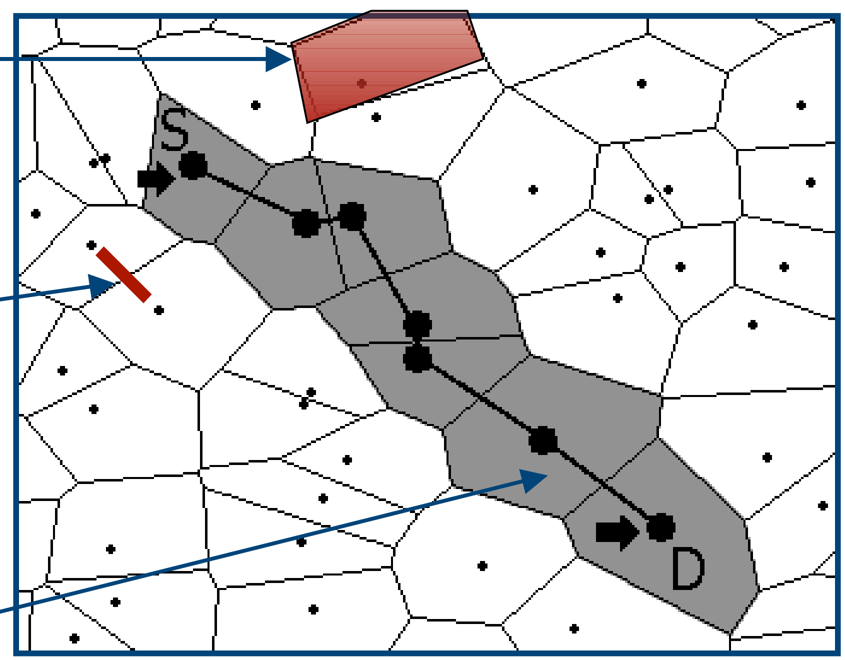
- ***This talk:*** attempt to systematically evaluate spatial energy burdens under <u>*proactive multipath routing*</u>

Modeling Ad hoc Network

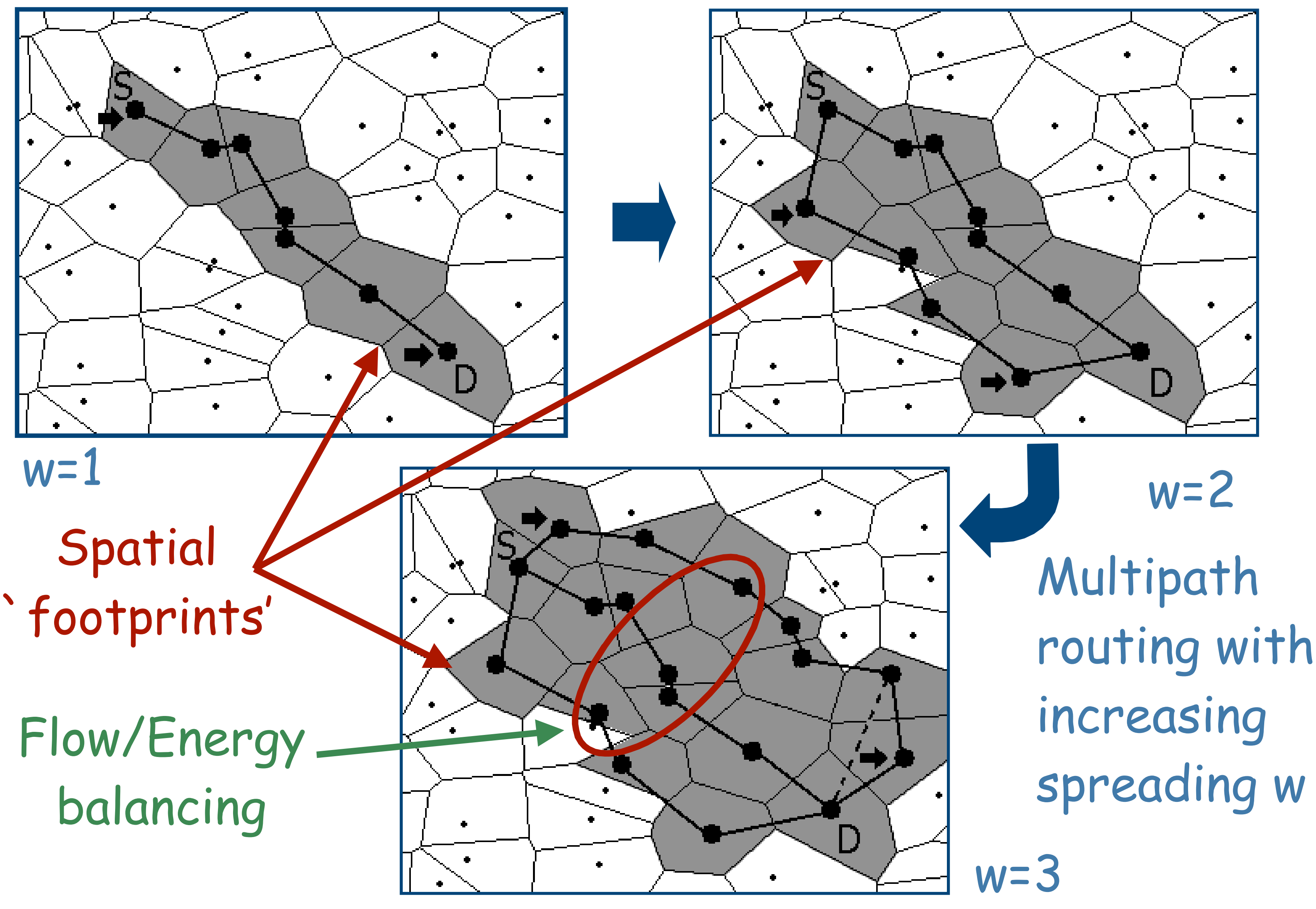
- Realization for node's locations:

$$\pi = \{x_i | i = 1, 2, \ldots\}$$

- **Voronoi tesselation:**
  - Each cell $V_{x_i}(\pi)$ is set of points which are closest to that node
- **Delaunay graph:** G(π, E)
  - E : edges placed between nodes whose cells share a face
- **Shortest Delaunay route:**
  - Shortest Eulclidean norm route on G(π, E)

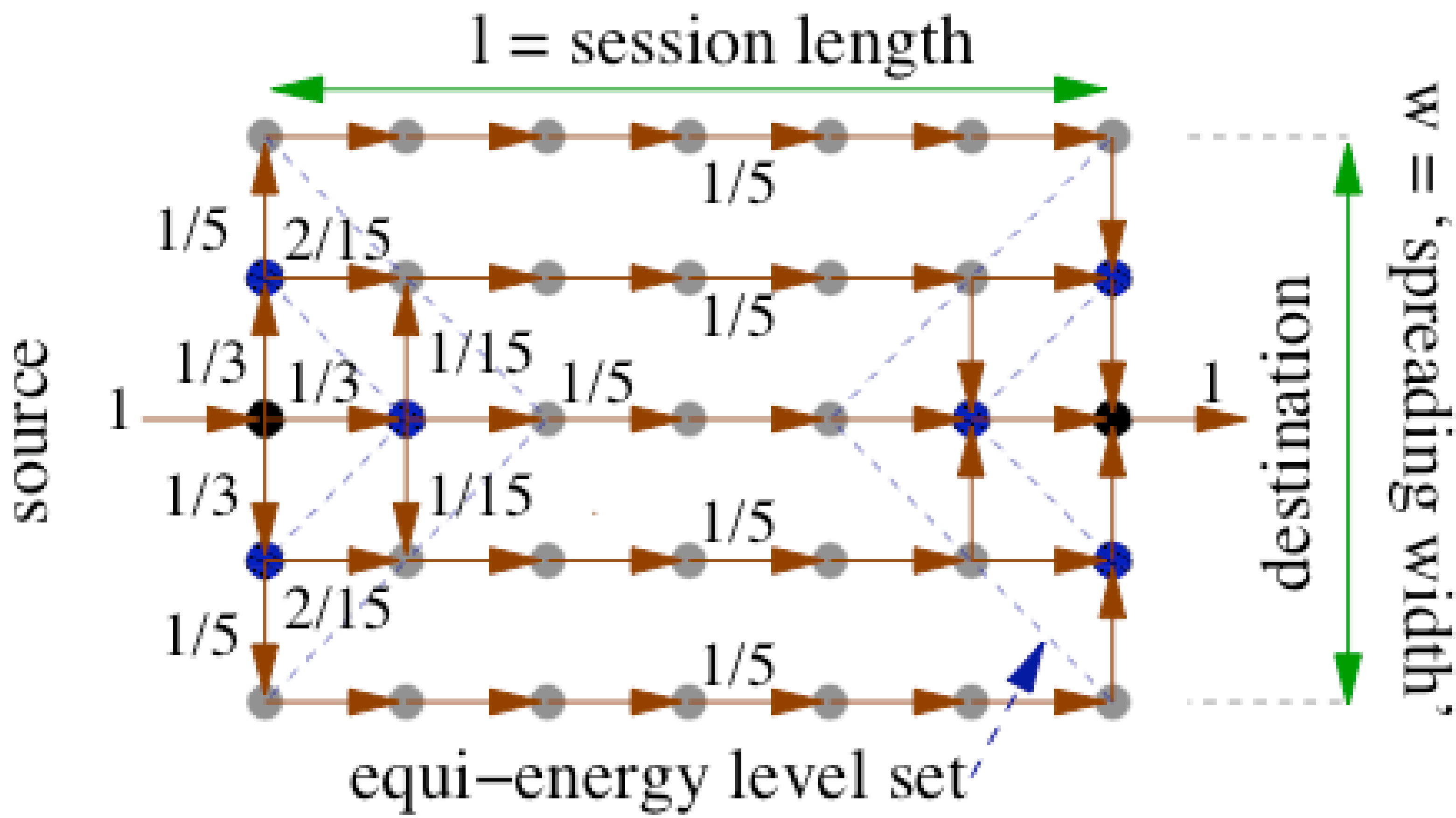


Multipath routing: geometric construction



Energy Balancing- Lattice Model

- Rectangular spatial footprint. Good flow balancing ?



Continuum Model

- Ad hoc nodes: infinitesimal units of space
- Traffic loads: random process of energy footprints
  - Session locations (offered load) prior to time t

$$\Pi_t = \{X_i | i = 1, 2, \ldots\} \sim \text{Poisson PP}(\lambda t)$$

  - Footprints- reflecting degree of spreading

$$\{\Phi_i | i = 1, 2, \ldots\} \quad \text{i.i.d. translation invariant}$$

  - Balancing of energy- reflecting flow across footprint

$$h(x, \Phi_i) \quad \text{energy burden density}$$

Continuum Model

- Cumulative energy burden - shot noise process

$$E(x,t) = \sum_{X_i \in \Pi_t} h(x - X_i, \Phi_i)$$

<u>Theorem</u>: (asymptotic normality)

$$\mu(t) = \mathbf{E}[E(0,t)] = \lambda t \mathbf{E}[\int_{\Phi_0} h(x, \Phi_0) dx]$$

$$\sigma^2(t) = \mathbf{Var}[E(0,t)] = \lambda t \mathbf{E}[\int_{\Phi_0} h(x, \Phi_0)^2 dx]$$

$$\frac{E(0,t) - \mu(t)}{\sigma(t)} \to N(0,1) \text{ as } t \to \infty$$

Evaluate impact of spreading mechanism on spatial energy burdens

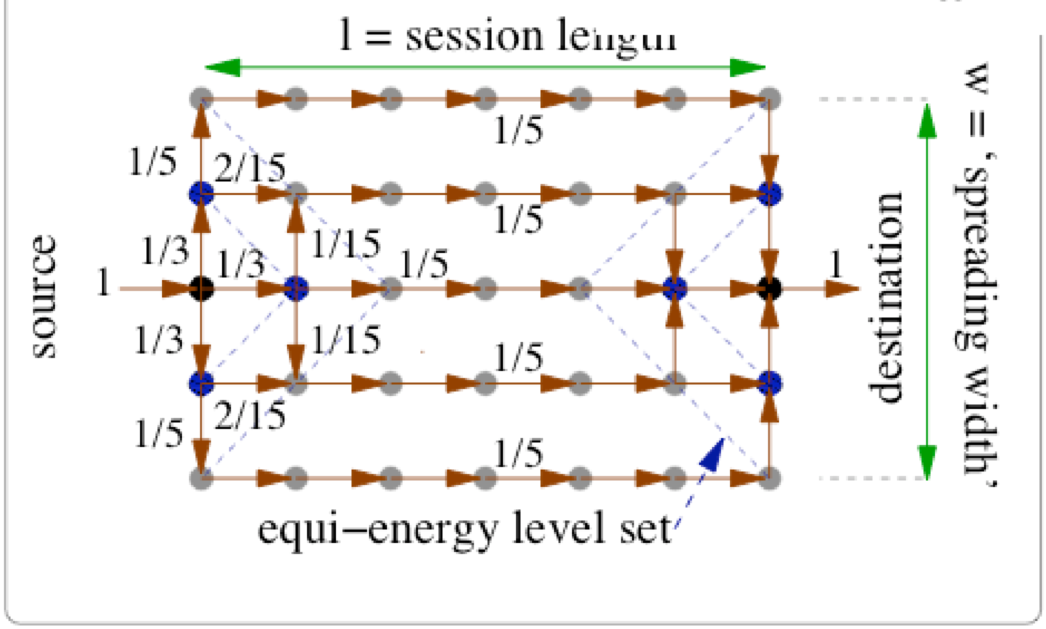
[Heinrich & Schmidt]

Lattice model: A parametrized energy balancing strategy

Lemma: min variance energy balancing strategy subject to flow conservation and equal flow on mid-section nodes of rectangular footprint gives:

$$\hat{\mu}(t) = \lambda t(l + \frac{w}{2} - \frac{1}{2w})$$

$$\hat{\sigma}^2(t) = \lambda t(\frac{l}{w} - \frac{1}{2}(1 + \frac{1}{w^2}) + \sum_{k=1}^{\frac{w+1}{2}} \frac{2}{2k-1})$$



Spreading Cost

Increasing w -> increasing mean & decreasing variance

Optimizing Energy Balancing:
Non replenishing case

- b = battery capacity   t= desired network lifetime
- Prob. of depletion by t for **typical** location/node

$$P(E(x,t) > b) \approx \phi(\frac{b-\mu(t)}{\sigma(t)}) = \phi(r(t))$$

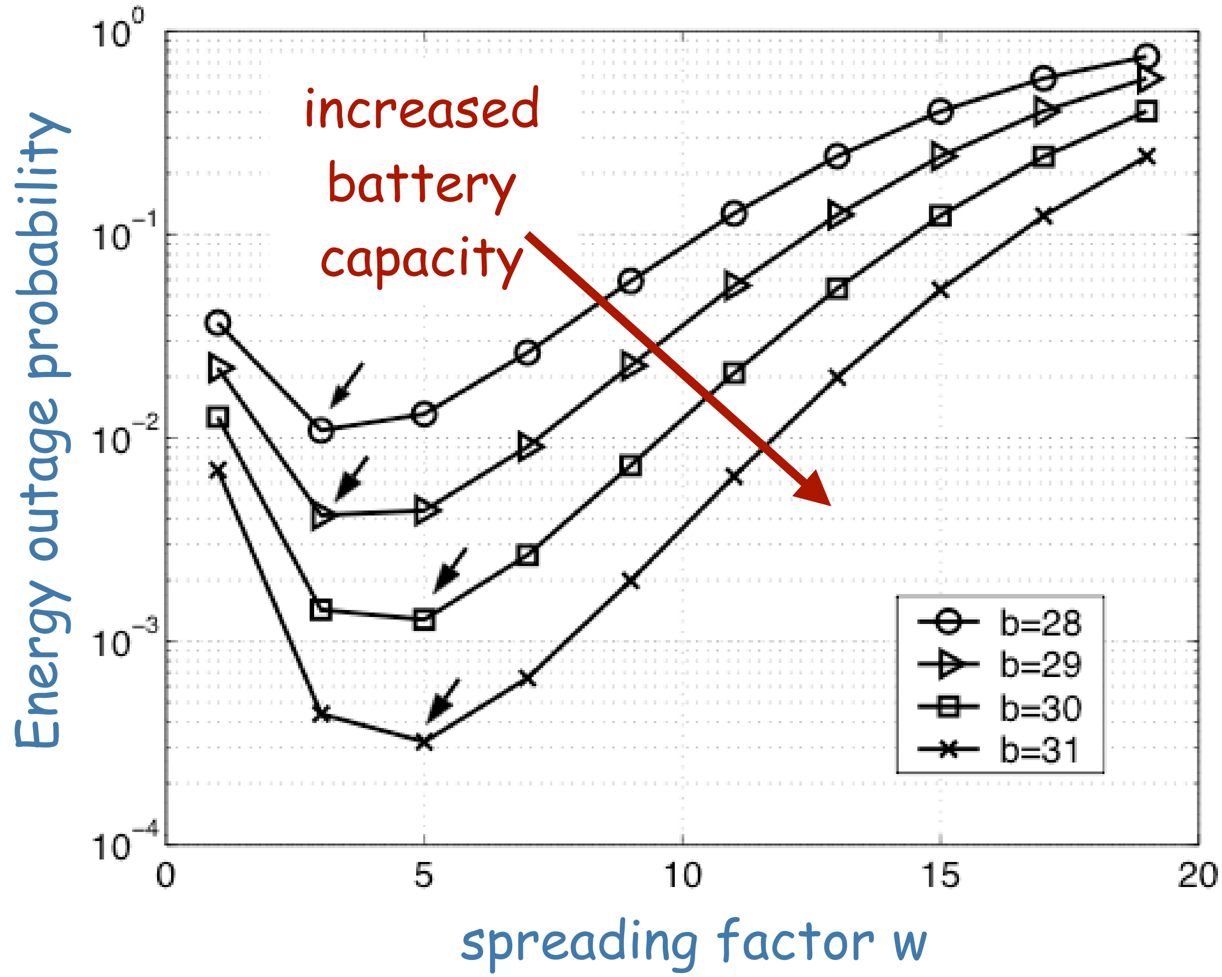
- Prob. of depletion by t of any location within A

$$P(\sup_{x\in A} E(x,t) > b) \approx H_\alpha a^{2/\alpha} r(t)^{4/\alpha} \phi(r(t))$$

α, a : depend on spatial correlations of energy burden field

[Adler,Aldous]

Optimizing non replenishing scenario: Example



- Larger battery capacity should increase spreading

Optimizing Energy Balancing:

Nodes with replenishing capability

- Simple discrete-time model with batch arrivals (of energy burdens) :

$$W_{n+1} = \max[W_n + S_n - c, 0]$$

Energy burden on node at the beginning of slot (n,n+1]

New energy burdens on node for slot (n,n+1]

Energy replenshing rate per slot

- How does energy balancing strategy impact tail asymptotics of stationary distribution ?

$$\lim_{b\to\infty} \frac{1}{b} \log P(W > b) = -\theta^*$$

Optimizing Energy Balancing:

Nodes with replenishing capability

- <u>Theorem:</u> Under our modelling assumptions if `energy queue' is stable, i.e.,

$$\lambda E[\int_{\Phi_0} h(x,\Phi_0)dx] < c$$

then the asymptotic tail exponent satisfies

$$\theta^* : \Lambda(\theta^*) = 0, \Lambda'(\theta^*) > 0$$

where

$$\Lambda(\theta) = \lambda E[\int_{\Phi_0} (e^{\theta h(x,\Phi_0)} - 1)dx] - c\theta$$

[Kelly, Whitt & Glynn, De Veciana & Walrand]

Optimizing network with replenishing
Example

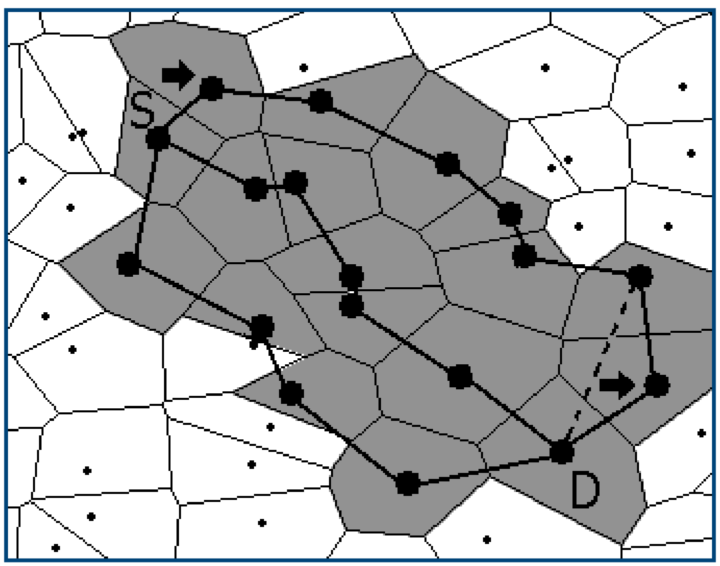
- Using grid model, l=8, λ=1 and c* is critical rate for w=7

Asympt. Decay rate θ^*		
Spreading factor w	Replenishing rates	
	c= 1.2 c*	c=2.0 c*
1	0.8673	1.7125
3	**1.2506**	2.7080
5	1.0965	**2.7593**
7	0.7965	2.6831

- Optimal tradeoff between maintaining stability and reducing energy burden variability.

Simulations: proactive multipath routing
Setup

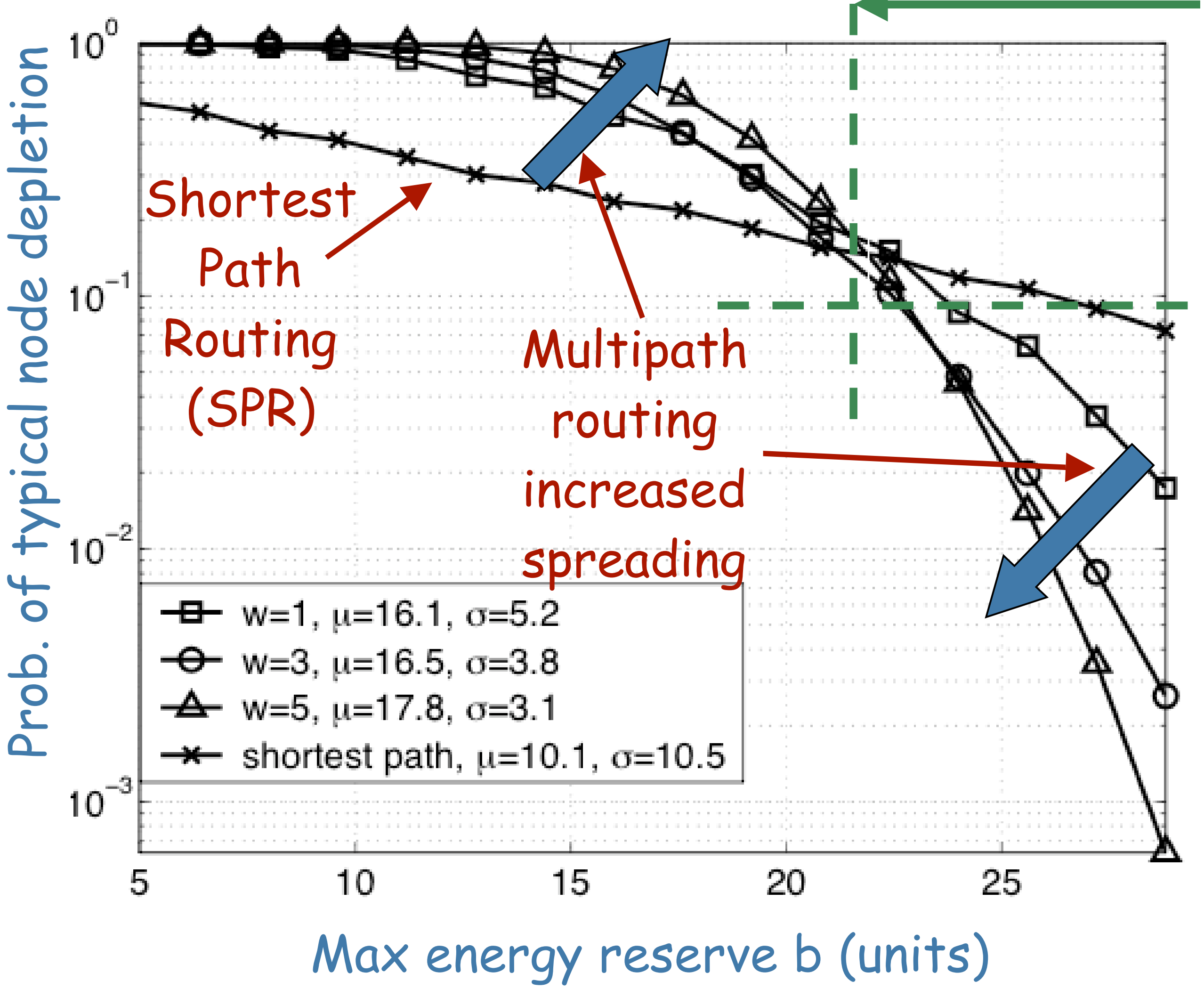
- 400 nodes locations on 20*20 square Poisson PP with unit rate
- Source-destination selected at random
- Multipath routing based on geometric construction for different spreading factors w



- Flow balancing mimics our optimal assignment
- Find probability that a randomly selected node is depleted of its energy reserve b

Simulations: Proactive multipath routing

Nonreplenishing case



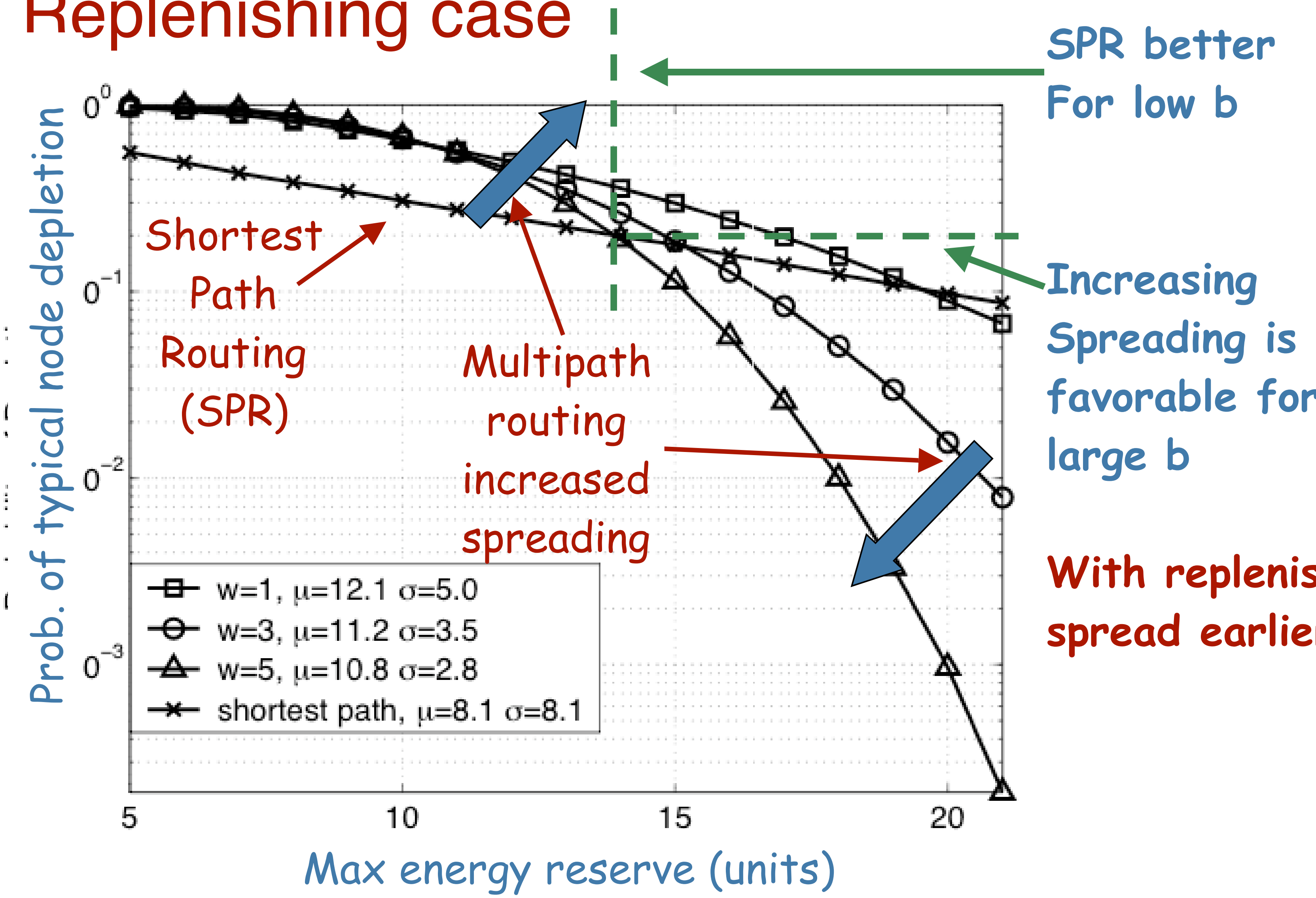
-SPR better for Low b

-SPR worse than SDR (w=1) for large b

Increasing spreading is favorable for large b

Simulations: Proactive multipath routing

Replenishing case



With replenishing spread earlier !

Summary and ongoing work - Part 1

- Investigate `optimal’ energy balancing strategy
  - Tradeoff: spreading to decrease variability versus energy cost of achieving spreading
- Stochastic geometric framework and simple queuing models enable study

- Ongoing
  - Continuum optimization - `optimal’ routing shape
  - Dynamic spreading based on stream characteristics
  - `Knock on’ effects in space - when to routing around depleted regions ?

Part 2: Routing hierarchies in wireless sensor networks using compression and sink nodes

- **Traffic and Network model**
  - Sensors generate stream of data packets which are routed via ad hoc network to set of sink nodes
  - Possibility of data/header compression of correlated/redundant data along intermediate nodes

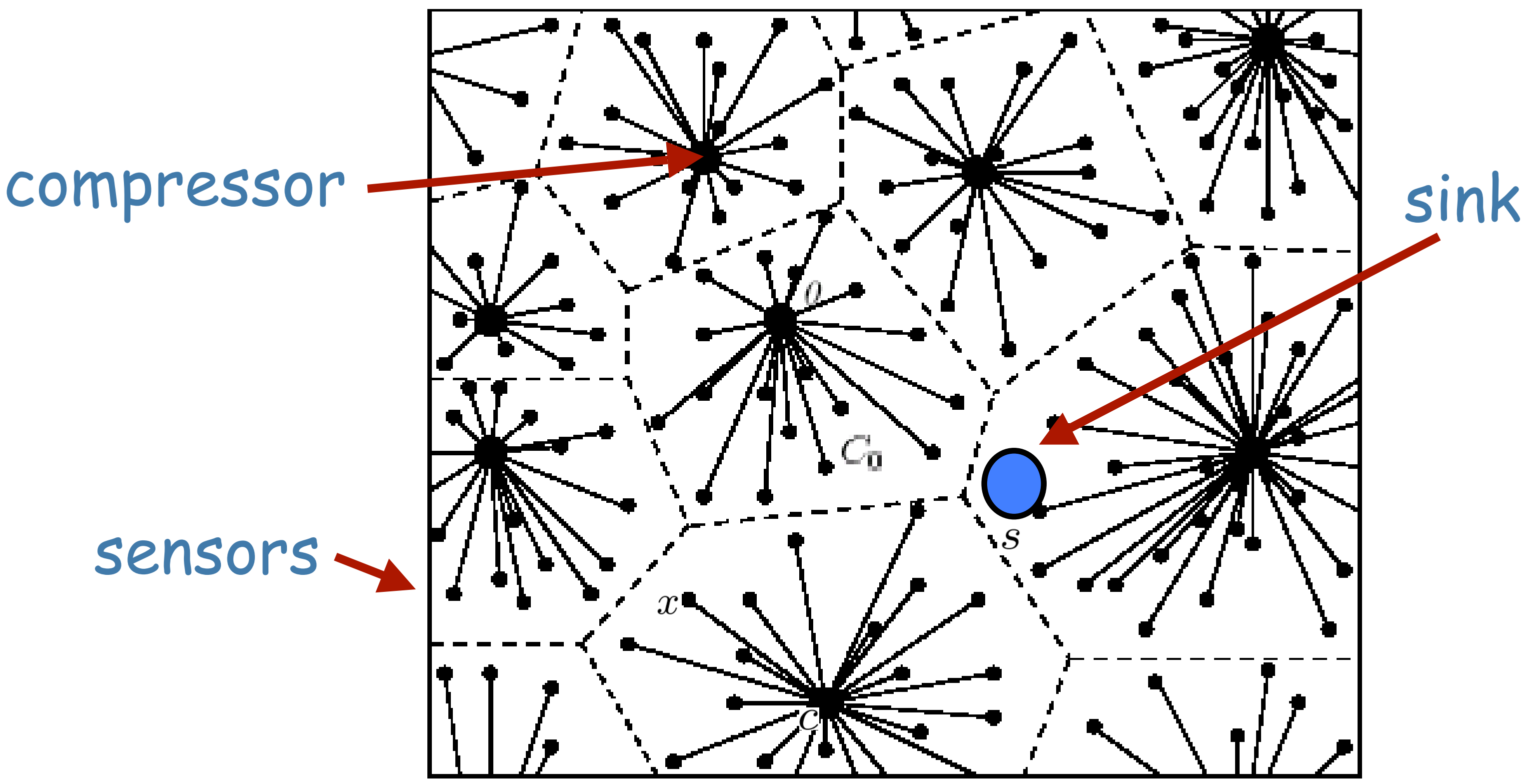
- **Problem:** What is the best way to organize compression & aggregation along with routing so as to minimize network's overall energy burden.

Hierarchical Network Organization

Model

- Locations of sensors, compressors, and sinks follow homogenous Poisson PPs

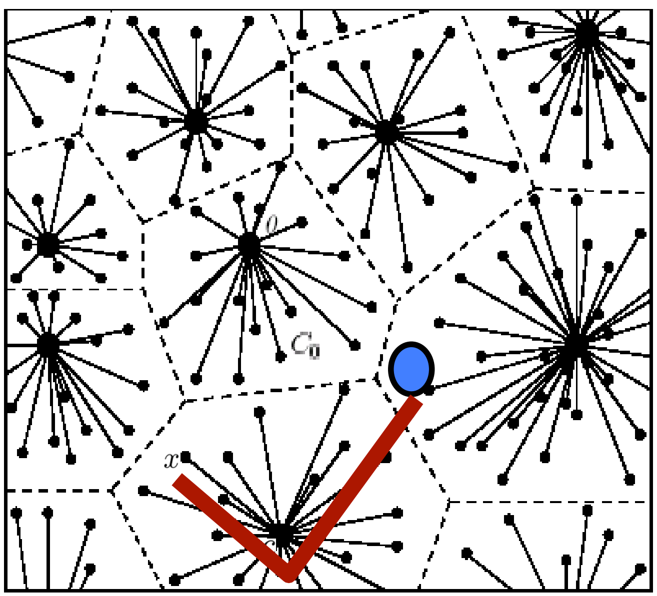
$$\Pi_0, \Pi_1, \Pi_2 \text{ with intensities } \lambda_0, \lambda_1, \lambda_2$$



Hierarchical Network Organization

Model

- Sensors generate packet rate at unit rate
- Energy cost between two locations is proportional to distance d(x,y) (I.e., ~ #of hops) and packet rate
- Compression ratio is roughly α



Energy cost e(x) for sensor at location x

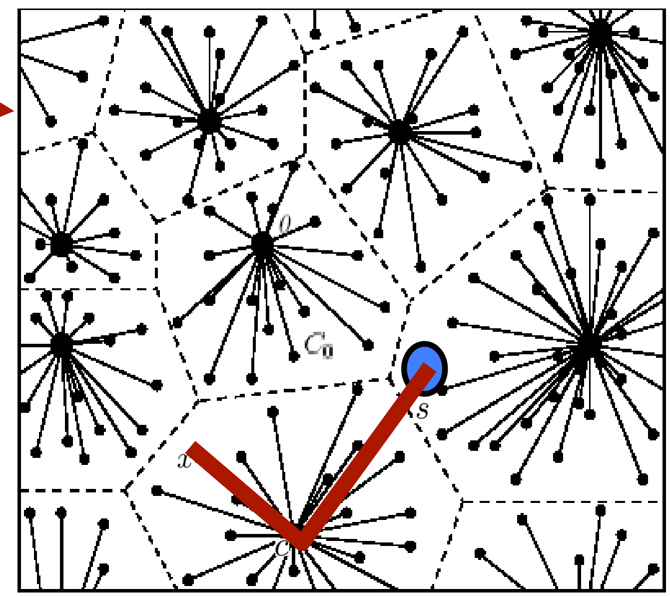
$$e(x) = d(x,c) + \alpha \cdot d(c,s)$$

Distance to compressor c

Distance from compressor to sink s

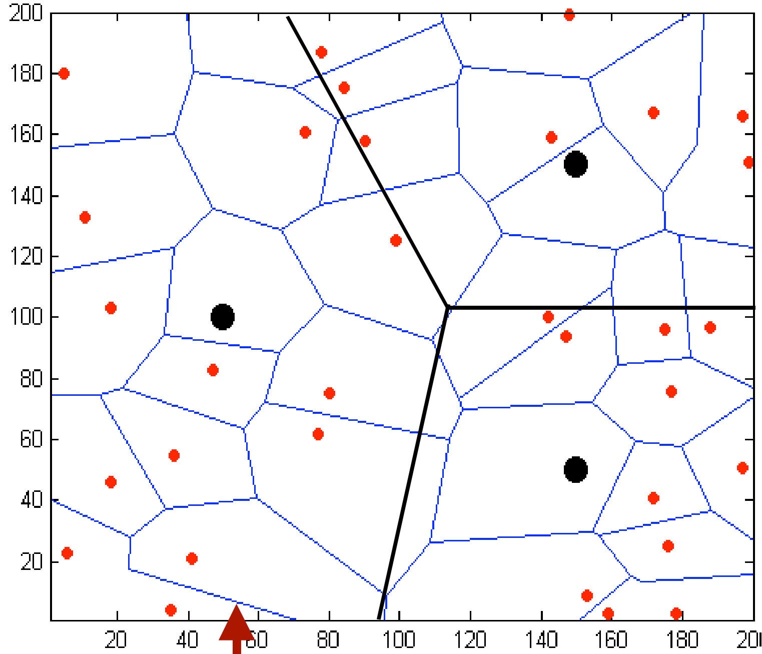
Hierarchical Network Organization

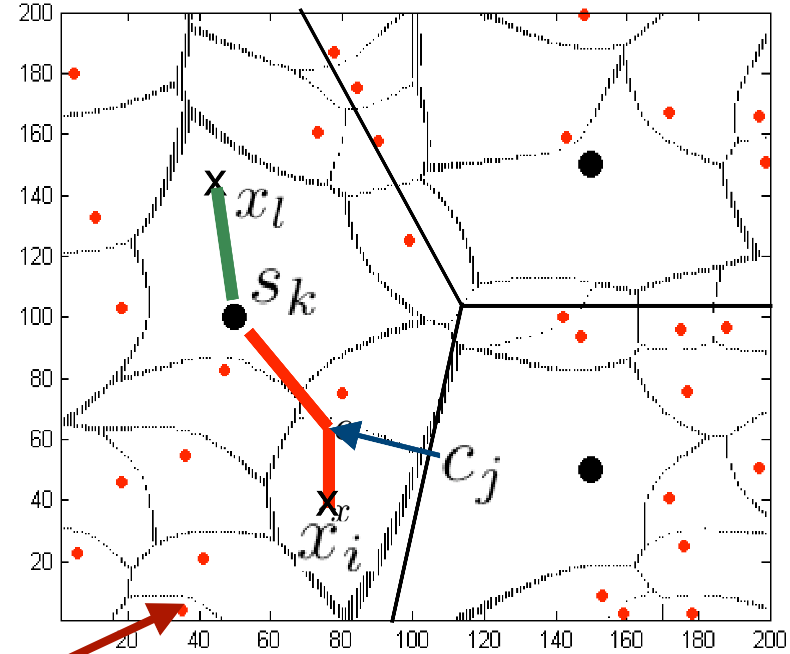
- **Problem:** What is routing/compression hierarchy which minimizes overall energy burden?
- Possible solution: route to closest compressor (or sink) and then from there to the sink
  - Voronoi tesselations *
  - But is this optimal?
- **Theorem:** The minimum avg. energy cost hierarchy is associated with a *Johnson-Mehl tessellation*



*[Baccelli et al.: Average cost analysis for Voronoi hierarchies]

Voronoi tessellation





● sink ● compressor (compression ratio $\alpha = 0.5$)

$$V_{y_i}(\pi_1, \pi_2) = \{z \in \mathbf{R}^2 \,||z - y_i| < |z - y_j|, \forall y_j \in \pi_1 \cup \pi_2\}$$

$$T_{y_i}(\pi_1, \pi_2) = \{z \in \mathbf{R}^2 \,||z - y_i| + \alpha|y_i - s_V(y_i)| < |z - y_j| + \alpha|y_j - s_V(y_j)|, \forall y_j \in \pi_1 \cup \pi_2\}$$

JM: compressor `seeds' start growing at times prop. to distance from closest sink - hyperbolic faces.

Analysis of energy cost

- Avg. cost for a typical sink:

$$E_2^0 \Bigg\{ \sum_{c_j \in \Pi_1 \cap V_0(\Pi_2)} \Bigg\{ \overbrace{\alpha |c_j| \mathcal{N}^{\alpha}_{c_j}}^{\text{compressor to sink}} + \overbrace{\sum_{x_i \in \Pi_0 \cap T^{\alpha}_{c_j}(\Pi_1, \Pi_2)} |x_i - c_j|}^{\text{sensors to compressor}} \Bigg\} \Bigg] + \underbrace{\sum_{x_i \in \Pi_0 \cap T^{\alpha}_{0}(\Pi_1, \Pi_2)} |x_i|}_{\text{sensors directly to sink at origin}} \Bigg\}$$

Expectation wrt Palm prob. sink process i.e. sink at origin

\# sensors associated with compressor c_j

Analysis of Energy Cost

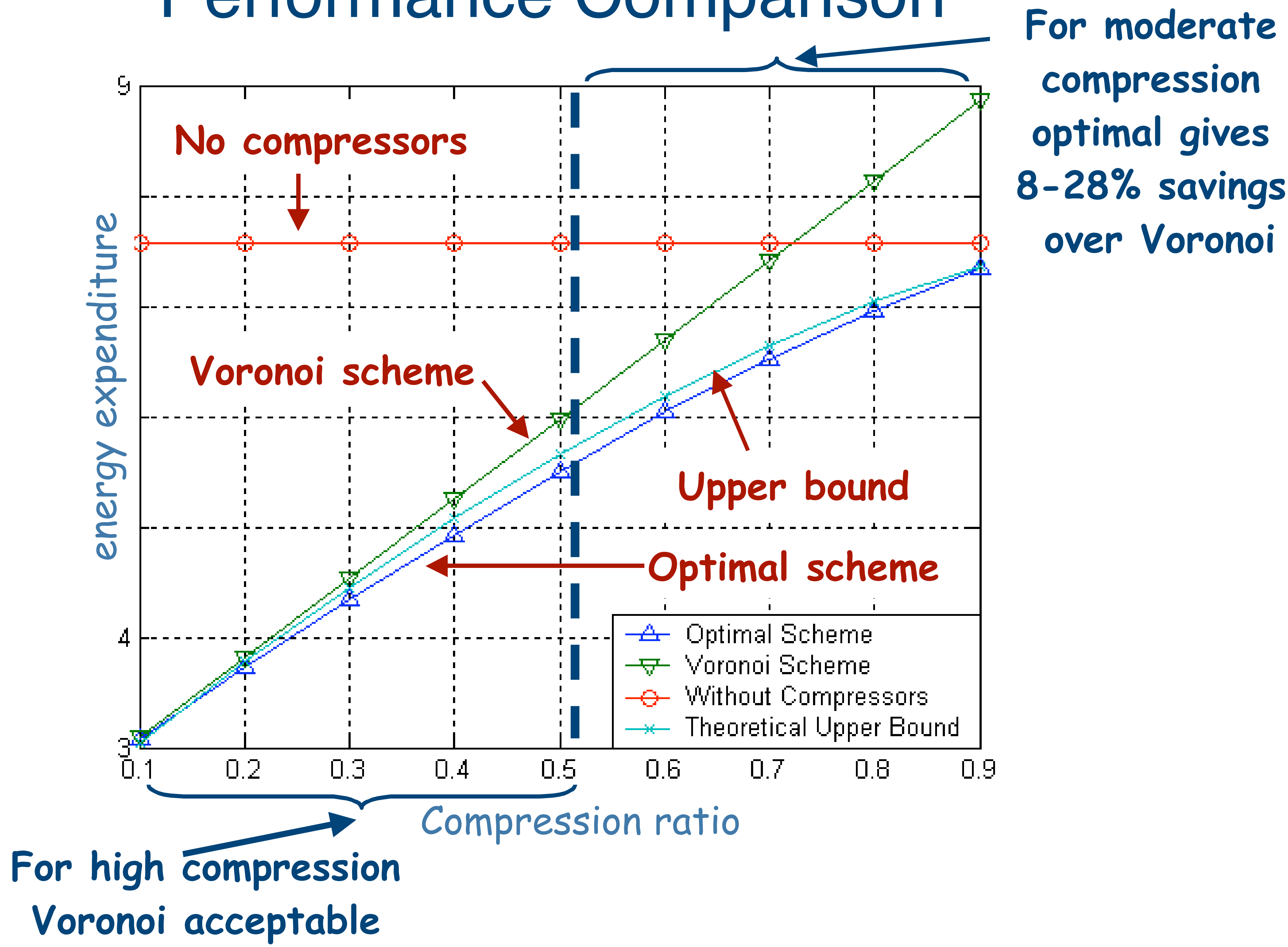
- **<u>Theorem</u>** a tight upper bound on energy cost is given by

$$\frac{\lambda_0}{\lambda_2}\left\{\frac{\alpha}{2\sqrt{\lambda_2}}+\frac{1-\alpha}{2\sqrt{\lambda_2+\frac{\lambda_1 f(\alpha)}{\pi}}}\right\}$$

$$f(\alpha) = \text{ area of } \alpha \text{ skewed Cartesian oval}$$

- α=0 -> avg. costs to closest sink/compressor $\dfrac{1}{2\sqrt{\lambda_1+\lambda_2}}$
- α =1 -> avg. costs to reach closest sink $\dfrac{1}{2\sqrt{\lambda_2}}$

Performance Comparison



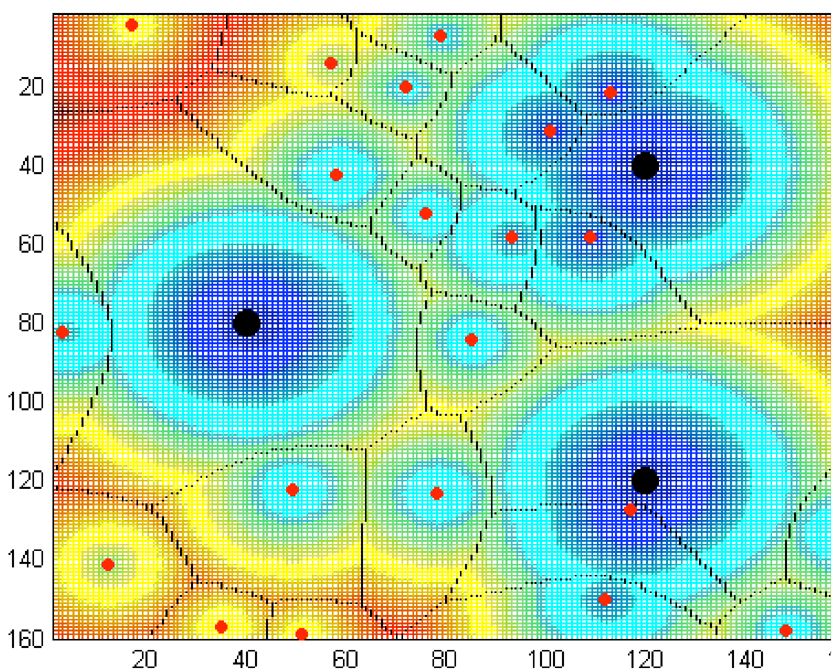
Results and extensions

- Analytical results for mean energy costs for optimal hierarchy associated with Johnson-Mehl tesselation
  - *Permit optimization of densities of compressors/ sinks etc. based an compression ratio and system capacity*

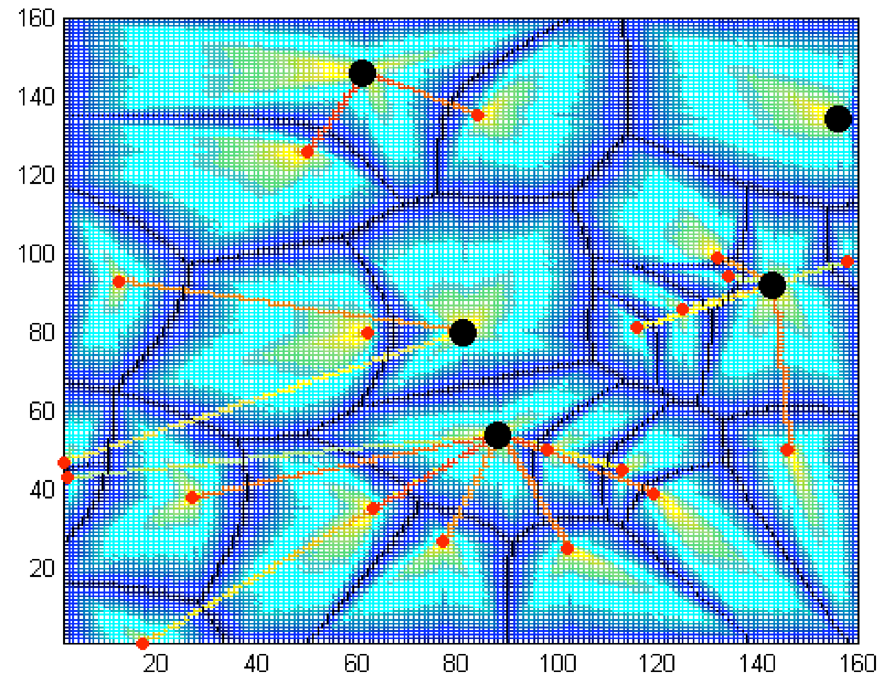
- Extensions: optimal hierarchy associated with non-linear energy costs
  - One/two hop model, i.e., direct transmission to sink or relaying via compressor to sink.
  - Capture wireless channel's signal decay (path loss)

Energy and `Congestion' Fields

compressor sink



Energy field associated with carrying traffic from given location e(x)



Cummulative energy field associated with ALL traffic traversing* given location with straight line routing

Summary - Part 2

- JM tessellation outperforms Voronoi scheme significantly when the density of compressors is fairly high, otherwise, *Voronoi scheme is as good as optimal scheme.*
- In one/two-hop cases, the gain from the optimal tessellation is much larger, however, as path-loss exponent increases, the role of compression becomes negligible
- *Congestion* is a severe impairment for the system design – detecting or switching compressors/sinks is unavoidable, but what is the best strategy? Further study

Outgoing comments.

- Stochastic geometry & queueing provides an concrete way to study spatial processes and interactions in ad hoc wireless and sensor networks.

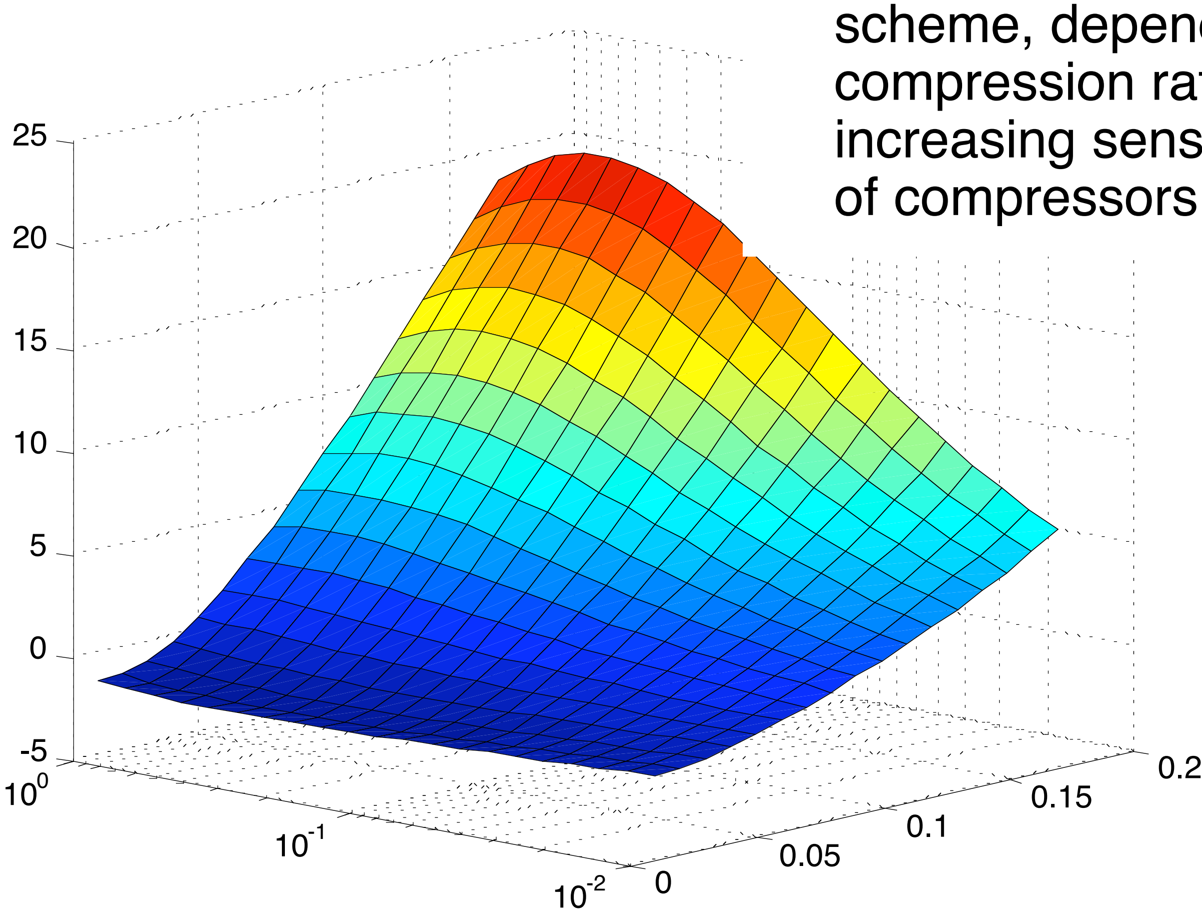
- Energy balancing-> optimal tradeoffs
  - Dynamic vs static settings, e.g., traffic/nodes

- We are looking to further refine these ideas and provide a more comprehensive view including some dynamic aspects of spatial interactions among user's traffic.

Spatial Dimension in Wireless and Sensor Networks

- Plays critical role in determining
  - Connectivity, capacity/interference patterns, energy expenditures, sensing coverage, protocol performance
  - Difficulties: complexity of environment, number of users/sensors, and mobility
- <u>Challenges</u>
  - Devise tools enabling modeling, analysis, and design of incorporating space/location
    - Macroscopic modeling via stochastic geometry
  - Develop more efficient system designs and optimized protocols

Performance Comparisons



- Energy savings of the optimal scheme, relative to Voronoi scheme, depends on the compression ratio with increasing sensitivity as density of compressors decreases