Exercises for Lecture #1

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1. Let X be an n-dimensional manifold and $N \in \mathbb{N}$, and let $S^m_{1,0} := S^m_{1,0}(X \times \mathbb{R}^N)$. Prove that

(i) any $m \in \mathbb{R}$, $S^m_{1,0}$ is a vector space w.r.t. pointwise linear combinations; and

(ii) for any m_1, $m_2 \in \mathbb{R}$, $a_j \in S^{m_j}_{1,0}$, $j = 1, 2 \implies a_1 \cdot a_2 \in S^{m_1+m_2}_{1.0}$.

2. Write $\mathbb{R}^n = \mathbb{R}^{n-k} \times \mathbb{R}^k$, with $x \in \mathbb{R}^n$ decomposing as $x = (x', x'')$. Consider the $(n-k)$-dimensional Lebesgue measure $\mu = dx'$ on $\mathbb{R}^{n-k}$ as a distribution on $\mathbb{R}^n$, $\langle \mu, f \rangle := \int_{\mathbb{R}^{n-k}} f(x', 0)\, dx'$, for $f \in \mathcal{D}(\mathbb{R}^n)$. Prove that $WF(\mu) = N^*\{x'' = 0\}$.

3. In $\mathbb{R}^3$, find the conormal bundles N^*Y of each of the two submanifolds,

(i) the sphere $Y_1 := \mathbb{S}^2 = \{x : |x| = 1\}$; and

(ii) the helix, $Y_2 := \{(\cos(t), \sin(t), t) : t \in \mathbb{R}\}$.