Exercises for Lecture #3

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1. Show that if $\phi(x, \theta)$ is a nondegenerate phase function, then the map

$$Crit_\phi \ni (x, \theta) \xrightarrow{j} (x, d_x\phi(x, \theta)) \in T^*X$$

is an immersion, i.e., has an injective derivative.

2. Suppose X and Y are smooth manifolds, not necessarily of the same dimension, and $Z \subset X \times Y$ is smooth. Let $\pi_X : Z \to X$ and $\pi_Y : Z \to Y$ be the coordinate projections to the left and right. Show that if

$$N^*Z \setminus \mathbf{0} \subset (T^*X \setminus \mathbf{0}) \times (T^*Y \setminus \mathbf{0}),$$

then π_X and π_Y are submersions.

3. The spherical mean operator T_4 is the lecture notes is an FIO associated with a **local** canonical graph which is no (global) canonical graph.

(i) Find the canonical relation C_1 to which T_4 is associated;

(ii) independently verify that C_1 is a canonical relation;

and (iii) verify that $C_1^t \circ C_1 \subseteq \Delta_{T^*\mathbb{R}^n} \cup C_2$, where C_2 is another canonical relation. What is the relationship between C_1 and C_2?